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Random Number Generator and Secure Communication Applications Based on Infinitely Many Coexisting Chaotic Attractors

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ABSTRACT

This paper aims to investigate a 3D chaotic system for applications on the secure communication system and random number generation. There exists a sinusoidal nonlinearity in the system making it uncommon of its type. Infinitely many chaotic attractors indicate multi-stability of the system, which is desired; for instance, the same system can be implemented for a multiple channel secure communication and switching between the channels can be achieved just by changing initial conditions. A brief mathematical analysis of the system is performed, and the circuit of the system is designed using active circuit elements. A synchronized system for secure communication is mathematically analyzed on MATLAB and simulated on PSPICE OrCAD. Synchronization of the system with the proposed circuit structure shows that this dynamic system can be used for chaotic communication. In addition, as an application of cryptography, a NIST* statistical test is performed on 10 bitstreams generated by the system. The bitstream produced has successfully passed all tests giving results in the length of the generated bit.

Keywords: Chaotic Circuit, chaotic masking, secure communication, random number generator

Introduction

Lorenz's [1] publication entitled "Does the flap of a butterfly's wings in Brazil set off a Tornado in Texas" was inspired by the basic characteristic of the chaotic systems. Lorenz observed extreme sensitivity of the chaotic systems to their initial conditions while performing weather simulations.

The fact that most of the real-world problems are nonlinear, with the discovery and the mathematical description of a simple chaotic system by Lorenz [2] has given the research on dissipative dynamical systems a new direction. Chaos theory, owing to its complex dynamics, has found applications in almost all the fields of life, including biology and medicine [3], management in industry [4], learning in classroom [5, 6], psychology [7], supply chain managements [8], and transportation [9, 10]. Chaos was thought undesirable and uncontrollable for some time but as the research moved from analysis to control of the chaotic systems [11-15], many novel chaotic systems are discovered and analyzed [16-21].

Cryptography is the combination of a Random Number Generator (RNG) and an encryption algorithm. Chaotic signals are applied widely in cryptography where random numbers are acting as fundamental units [23-29]. Demir and Ergün [30] performed analysis of deterministic chaos, generated by a phase-locked loop device and showed that unbounded chaos and Gaussian white noise are comparable as an entropy source; the same authors demonstrated some security vulnerabilities of the cryptography (RNG and XOR encryption algorithm) based on a 4D chaotic hyperjerk system [31]. Tutueva et al. [32] proposed Adaptive Zaslavsk web maps for generation of pseudorandom numbers (PRN) to improve chaos-based cryptography. Another efficient digital image encryption algorithm utilizing PRN generator, based on chaotic fractal sequence was proposed by Ayubi et al. [33]. Sprott and Thio [34]generated Gaussian Random Numbers with a simple chaotic circuit.

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Chaotic signals have broadband and noise-like spectrum; hence, chaotic systems synchronize only with their selves [35] and are very practical for communication systems. After the proof of experimental feasibility of the chaotic systems for communication [15], many chaotic systems are synchronized using different synchronization algorithms. Sundarapandian and Pehlivan [17], achieved adaptive synchronization of a novel chaotic system with its realization in PSpice. Sundarapandian et al. [21] applied a new complex 3D chaotic system on an autonomous wireless mobile robot and realized its circuit on MultiSIM10. Herman Sedef and others synchronized and simulated the Lorenz system with two different methods [36-38].

In this study, a recently reported chaotic system having infinitely many coexisting attractors is a matter of concern for RNG and secure communication system [22]. Synchronization in this study is achieved by the addition of a difference term, a method introduced by Sambas et al. [38]. Random 10 bitstream is generated by performing XOR operation on two different signals of the system, and the randomness of the generated sequence is proved with NIST statistical test [39].

System

In this study, an extremely simple chaotic system with infinitely many coexisting chaotic attractors, which is defined as in (1) consisting of five terms with two nonlinearities, has been analyzed and applied for RNG and secure chaotic communication [22].

$$\dot{x}_1 = a(x_2 - x_1) \tag{1a}$$

$$\dot{x}_2 = bx_1 \sin(x_3) \tag{1b}$$

$$\dot{x}_3 = c - x_1 x_2 \tag{1c}$$

x_1, x_2, and x_3 are state variables and a, b, and c are positive constants. Two nonlinearities of the system are x_1 and x_1x_2, respectively.

Mathematical Analysis

Jacobian matrix, Lyapunov exponents, bifurcation diagram, phase portrait, and time series analysis are some of the important methods to verify and control the chaoticity of any system. These methods are briefly studied. Sinusoidal nonlinearity of the system (1) indicates an infinite number of unstable equilibria.

by $\dot{x}_1 = \dot{x}_2 = \dot{x}_3 = 0$, resulting equilibrium of the system (1)

$$(1a) \Rightarrow x_2 = x_1$$

$$(1b) \Rightarrow x_3 = sin^{-1}(0) = k\pi \qquad \text{where } k = 0, \pm 1, \pm, \pm 3, \cdots$$

$$(1c) \Rightarrow x_1 x_2 = c \quad \Rightarrow \quad x_2 = x_1 = \pm\sqrt{c}$$

$O = \{(\dot{x}_1, \dot{x}_2, \dot{x}_3) \mid x_2 = x_1 = \pm\sqrt{c}, x_3 = k\pi\}$.

System (1) is multistable because of the coexistence of attractors and its final state is decided by initial conditions [40]. Jacobian matrix gives vital information for the analysis of a chaotic system.

$$J = \begin{bmatrix} \frac{\partial \dot{x}_1}{\partial x_1} & \frac{\partial \dot{x}_2}{\partial x_1} & \frac{\partial \dot{x}_3}{\partial x_1} \\ \frac{\partial \dot{x}_1}{\partial x_2} & \frac{\partial \dot{x}_2}{\partial x_2} & \frac{\partial \dot{x}_3}{\partial x_2} \\ \frac{\partial \dot{x}_1}{\partial x_3} & \frac{\partial \dot{x}_2}{\partial x_3} & \frac{\partial \dot{x}_3}{\partial x_3} \end{bmatrix} = \begin{bmatrix} -a \; a & 0 \\ b\sin(x_3) \; 0 & bx_1 \cos x_3 \\ -x_2 \; -x_1 & 0 \end{bmatrix}$$

resulting gradient of system $\nabla V = \frac{\partial \dot{x}_1}{\partial x_1} + \frac{\partial \dot{x}_2}{\partial x_2} + \frac{\partial \dot{x}_3}{\partial x_3} = -a.$

Gradient $\nabla V < 0$ is a necessary condition for any system to be chaotic. Therefore, for any positive value of "α", system (1) will show chaotic behavior given suitable initial conditions and its phase space will become wider continuously [41].

Bifurcation diagram shows the ranges of a dynamical system for chaoticity, periodicity, and quasi periodicity, under the condition that one of the parameters (bifurcation parameter) is not fixed. By utilizing this information, we can generate the desired behavior of our system. The bifurcation diagram of system (1) with respect to the parameter c ∈ (0, 1) taking α = 1 and b = 12 is shown in Figure 1. The system (1) is periodic for a wide range of c>0.7, chaotic for 0<c<0.55, and quasi periodic for 0.55<c<0.7 approximately.

Lyapunov exponents or Lyapunov characteristic exponents of a dynamical system are the rates of convergence or divergence of infinitesimally nearby orbits. Lyapunov exponent for constant parameters of α = 1, b = 12, and c = 0.30 (chaotic range) is resulting (+,0,-) exponents (Figure 2a) and for c=1 (periodic range) is resulting (0,-,-) exponents (Figure 2b), given initial conditions of [1, 0, 1]. In the prior case, the largest exponent is positive, verifying the chaoticity and the sensitivity of system (1) to initial conditions [42, 44]. In the latter case, the largest Lyapunov exponent is zero so the system will become independent of the initial conditions over time [43].

3D projections of phase space are observed for the initial conditions of $\{x_2 = x_1 = 1, x_3 = s\pi \mid s = 2, 4, 6, 8, 10, 12\}$ and with the constant parameters a = 1, b = 12. Closed curves for c = 1 in Figure 3a represent periodic systems, whereas complex shapes for c = 0.3 in Figure 3b are of chaotic systems.

Finally, the chaoticity of the system is verified by analysis of the time series. Analyzing a system in the time domain is an easy way to determine whether a system is periodic or not. It can be verified from Figure 4a that for c = 1, system is in the periodic region and for c = 0.3, system is in chaotic region Figure 4b.

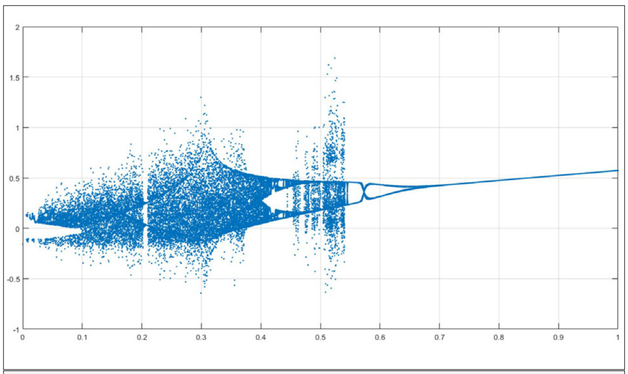

Figure 1. Bifurcation diagram for c ∈ (0, 1), a = 1 and b = 12

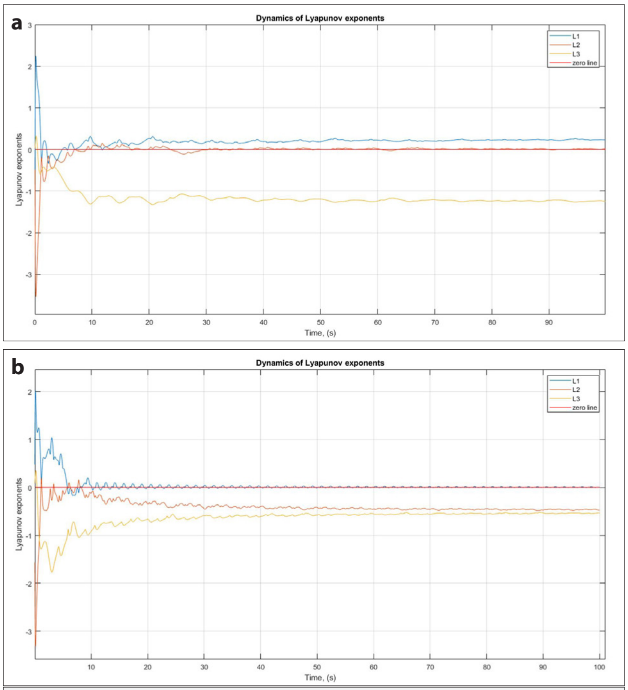

Figure 2. a, b. Chaotic state Lyapunov exponents (a). Periodic state Lyapunov exponents (b)

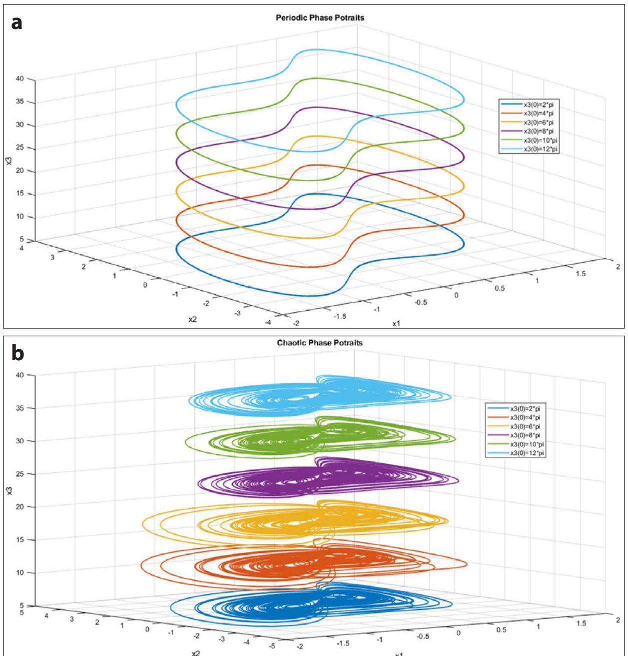

Figure 3. a, b. Periodic state phase portraits (a). Chaotic state phase portraits (b)

Circuit Design Using Active Elements

Parallel synthesis method using op amp circuits have been used for circuit realization in past studies. However, a higher number of active and passive elements have many disadvantages including higher power consumption, system complexity, and difficulty to

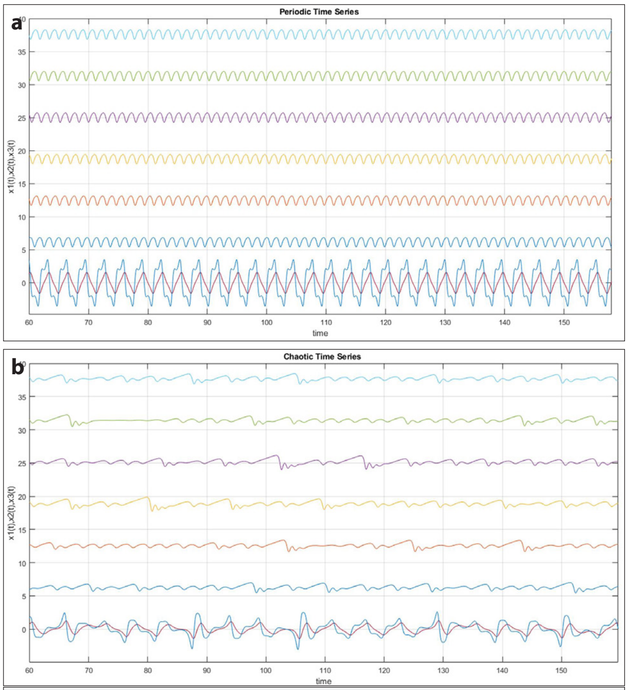

Figure 4. a, b. Periodic time-series for {$x_2=x_1=1, x_3= s\pi$ |s=2, 4, 6, 8, 10, 12} (a). Chaotic time-series for {$x_2=x_1=1, x_3= s\pi$ |s=2, 4, 6, 8, 10, 12} (b)

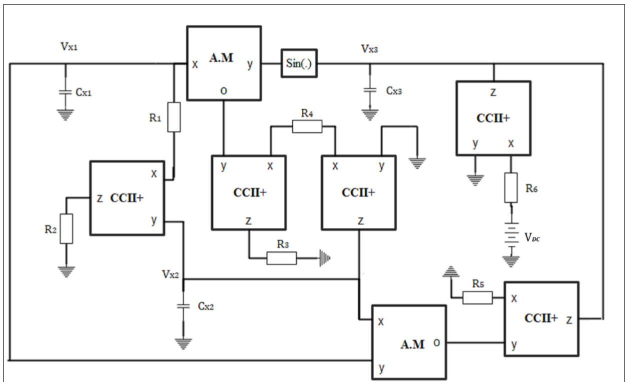

Figure 5. Proposed circuit diagram with CCII+ and AM

control. Therefore, to avoid these problems, circuit of the system (1) is designed using alternative active elements i.e., second generation current conveyors (CCII+) and analog multipliers (AM). The circuit diagram of the system is shown in Figure 5. Using the proposed circuit, the number of active elements was reduced from 9 to 7 and the number of passive elements was reduced from 12 to 9.

By applying basic circuit analysis on the circuit diagram, system equations are derived as (2).

Kirchhoff's Current Law at node Vx1

$$c_{x_1}\frac{dv_{x_1}}{dt} = \frac{v_{x_2} - v_{x_1}}{R_1} \Rightarrow \frac{dx_1}{dt} = \frac{(v_{x_2} - v_{x_1})}{R_1 * c_{x_1}} \qquad (2.1)$$

Kirchhoff's Current Law at node Vx2

$$c_{x_2}\frac{dv_{x_2}}{dt} = \frac{kv_{x_1}\sin v_{x_3}}{R_4} \Rightarrow \frac{dv_{x_2}}{dt} = \frac{kv_{x_1}\sin v_{x_3}}{R_4 * c_{x_2}} \quad (2.2)$$

Kirchhoff's Current Law at node Vx3

$$c_{x_3}\frac{dv_{x_3}}{dt} = \frac{V}{R_6} - \frac{kv_{x_1} * v_{x_2}}{R_5} \Rightarrow \frac{dv_{x_3}}{dt} = \frac{V}{c_{x_3} * R_6} - \frac{kv_{x_1} * v_{x_2}}{c_{x_3} * R_5} \quad (2.3)$$

By assuming $\frac{dv_{x_1}}{dt} = \dot{x}_1, \frac{dv_{x_2}}{dt} = \dot{x}_2, \frac{dv_{x_3}}{dt} = \dot{x}_3, v_{x_1} = x_1, v_{x_2} = x_2, v_{x_3} = x_3, a = \frac{1}{R_1 C_{x_1}},$ $b = \frac{k}{c_{x_2} * R_4}$ $c = \frac{V}{R_6 C_{x_3}}$ and $\frac{k}{c_{x_3} * R_5} = 1,$, it can be proved that system (2) is equivalent to system (1).

Values of the circuit elements, calculated by assuming a = 1, b = 12, c = 0.3, and k = 0.1 are given in Table 1. DC voltage source *VDC = 30 mV* for constant parameter c.

Table 1. Calculated values of circuit elements

Element	R_1	R_4	R_5	R_6
Value	5 *kΩ*	5 *kΩ*	5 *kΩ*	5 *kΩ*
Element	C_{x_1}	C_{x_2}	C_{x_3}	*k*
Value	200 *μ*F	1.667 *μ*F	20 *μ*F	0.1 V^{-1}

To operate the system in realizable range amplitude, scaling coefficient K_a and frequency scaling coefficient K_f are taken as 0.01 and 1000, respectively, resulting in denormalized elements values of $R = 50$, $Cx_1 = 20\mu F$, $Cx_2 = 0.1667\mu F$, and $Cx_3 = 2\mu F$. Pspice macro-models of commercially available integrated circuits are used for PSpice simulation. AD633 [44] integrated circuit is used for AM, whereas AD844 [45] is used for CCII+ by keeping w port as an open circuit. Comparison of 2D phase portraits and time series generated from the mathematical analyzes in MATLAB and simulation in PSpice are given in Figure 6 and Figure 7, respectively.

Chaotic Synchronization

As a practical application, the communication system is synchronized by adding a difference term; this method presented in [38]. Below are the system equations of the transmitter (3), the receiver (4), and message signal (5) derived from the circuit diagram of the communication system given in Figure 8. The advantage of this method over the other widely used methods [37] is that the synchronization signal does not need to be sent continuously. As the difference term in (4.3) will be zero after the systems are synchronized, transfer of the synchronization signal x_3 can be stopped, thus continuing communication with only one channel.

$$\dot{x}_1 = \frac{(x_2 - x_1)}{R_1 * c_{x_1}} \quad (3.1)$$

$$\dot{x}_2 = \frac{kx_1 \sin x_3}{R_4 * c_{x_2}} \quad (3.2)$$

$$\dot{x}_3 = \frac{V}{c_{x_3} * R_6} - \frac{kx_1 x_2}{c_{x_3} * R_5} \quad (3.3)$$

$$\dot{x}_4 = \frac{(x_5 - x_4)}{R_1 * c_{x_1}} \quad (4.1)$$

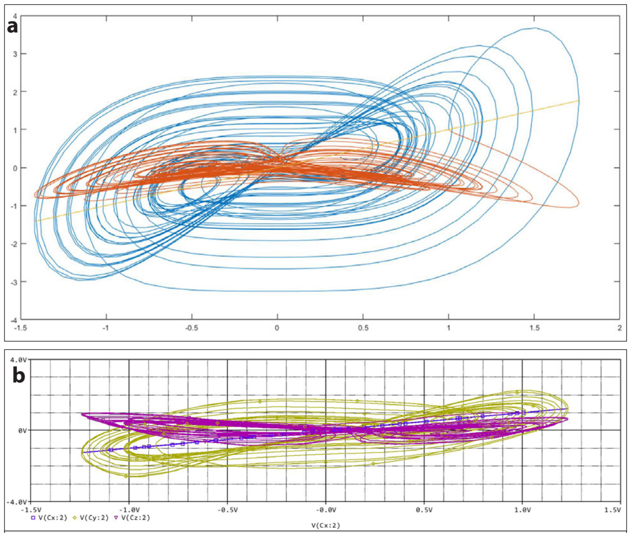

Figure 6. a, b. Phase Portrait obtained mathematically in MATLAB (a). Phase Portraits obtained with simulation in PSpice (b)

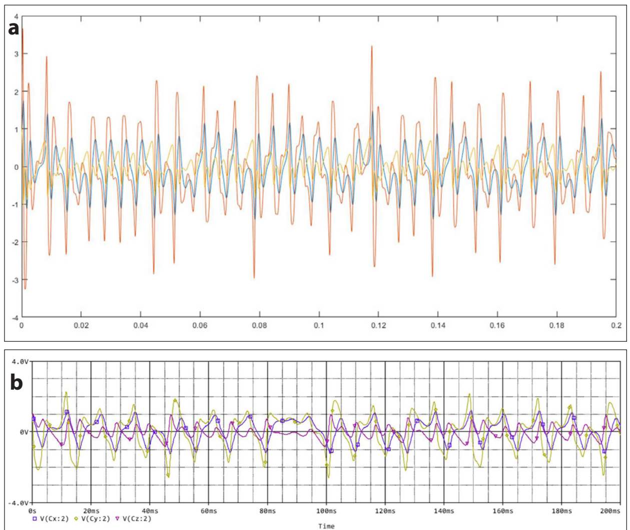

Figure 7. a, b. Time-series obtained in MATLAB (a). Time-series obtained with simulation in PSpice (b)

$$\dot{x}_5 = \frac{kx_4 \sin x_6}{R_4 * c_{x_2}} \quad (4.2)$$

$$\dot{x}_6 = \frac{V}{c_{x_3} * R_6} - \frac{kx_4 x_5}{c_{x_3} * R_5} + \frac{(x_3 - x_6)}{R_7 * c_{x_3}} \quad (4.3)$$

$$\dot{m} = -A * \omega * \sin(\omega t) \quad (5)$$

In this system, after the receiver and transmitter are synchronized by using x_3 signal, namely V_z, the information signal to be transmitted is added to x_1, V_x on the receiver side, and sent to the receiver through the communication channel. As the V_x signal is produced exactly on the receiver side (as x_4), the original information message (*m(t)*) is obtained by subtracting the V_x on the receiver side from the received signal. The block diagram of the method is given in Figure 9.

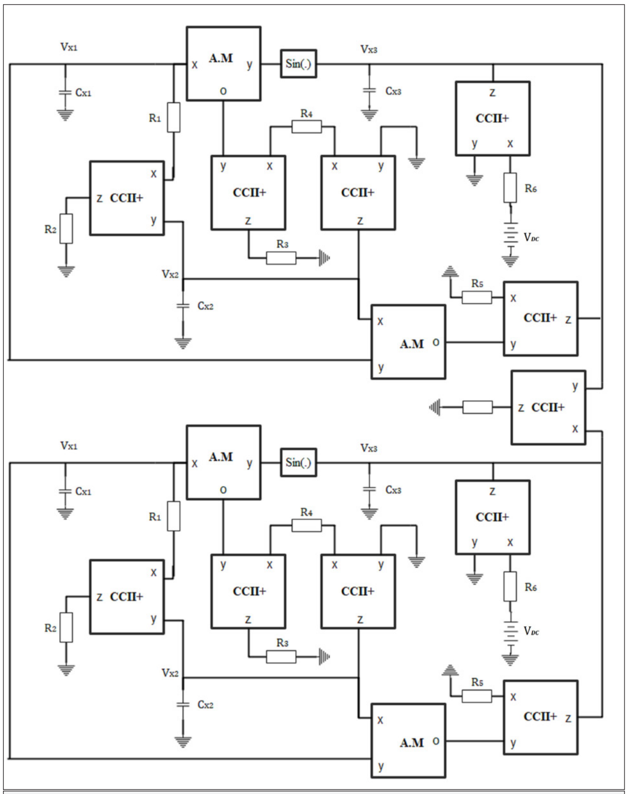

Figure 8. Synchronization of transmitter and receiver circuits by using difference adding method

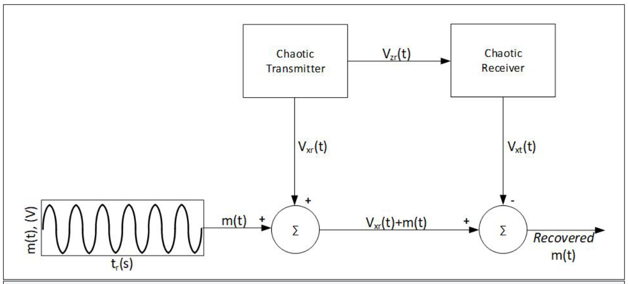

Figure 9. Chaotic Communication block diagram

By setting ω=2*pi*40 and value of difference term coefficient as -300, the system gets synchronized after approximately 120 ms in MATLAB and it takes approximately 160 ms in PSpice. Comparison of received versus transmitted V_z (x_3 and x_6) drawn by MATLAB and PSpice simulation is provided in Figure 10a and Figure 10b, respectively. Initial conditions for transmitter circuit are (1,1,1) and for receiver circuit are (1,1,0). Comparison of transmitted and received messages plotted in MATLAB is provided in Figure 11a and 11b.

Furthermore, the system is synchronized using another method, signal transfer [35]. x_2 is selected as the signal to be transferred, and V_x (x_1 and x_4) signals initiated from different initial conditions are observed. Initial conditions for transmitter circuit are (1,1,1) and for receiver circuit are (0,1,1.2). Equations

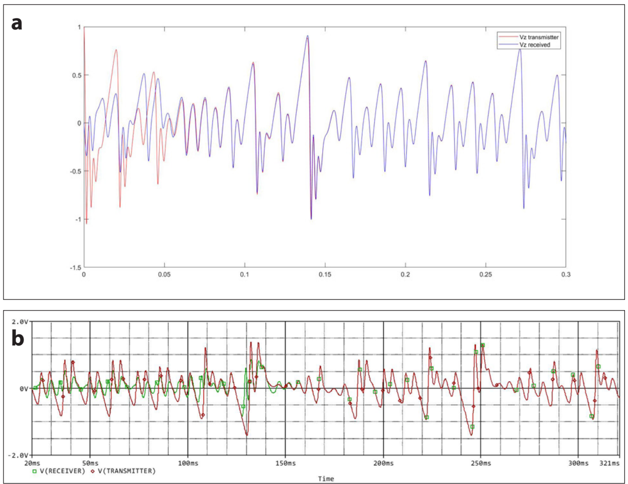

Figure 10. a, b. Transmitter and Receiver Vz in MATLAB for difference adding method (a). Transmitter and Receiver Vz in PSpice OrCAD for difference adding method (b)

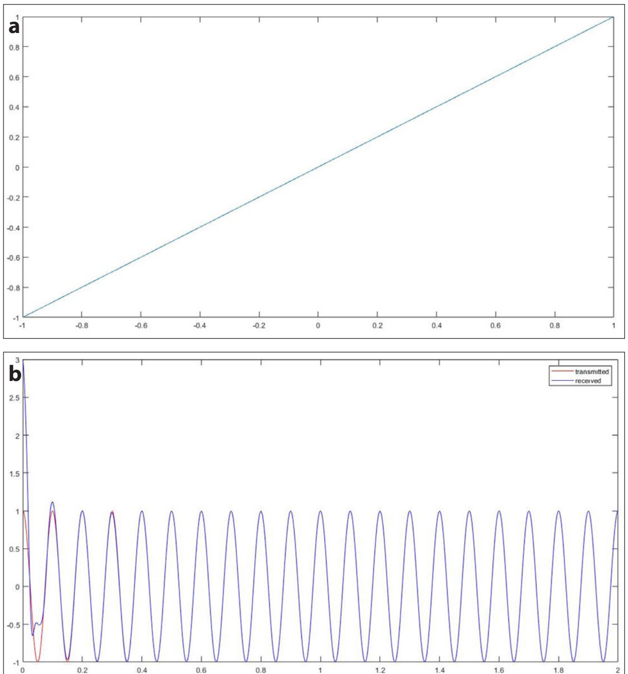

Figure 11. a, b. Transmitted vs. received message (a). Transmitted and received signal in time domain (b)

used for receiver and transmitter circuits are given in (6) and (7), respectively.

$$\dot{x}_1 = \frac{(x_2 - x_1)}{R_1 * c_{x_1}} \quad (6.1)$$

$$\dot{x}_2 = \frac{k x_1 \sin x_3}{R_4 * c_{x_2}} \quad (6.2)$$

$$\dot{x}_3 = \frac{V}{c_{x_3} * R_6} - \frac{k x_1 x_2}{c_{x_3} * R_5} \quad (6.3)$$

$$\dot{x}_4 = \frac{(x_2 - x_4)}{R_1 * c_{x_1}} \quad (7.1)$$

$$\dot{x}_5 = \frac{k x_4 \sin x_6}{R_4 * c_{x_2}} \quad (7.2)$$

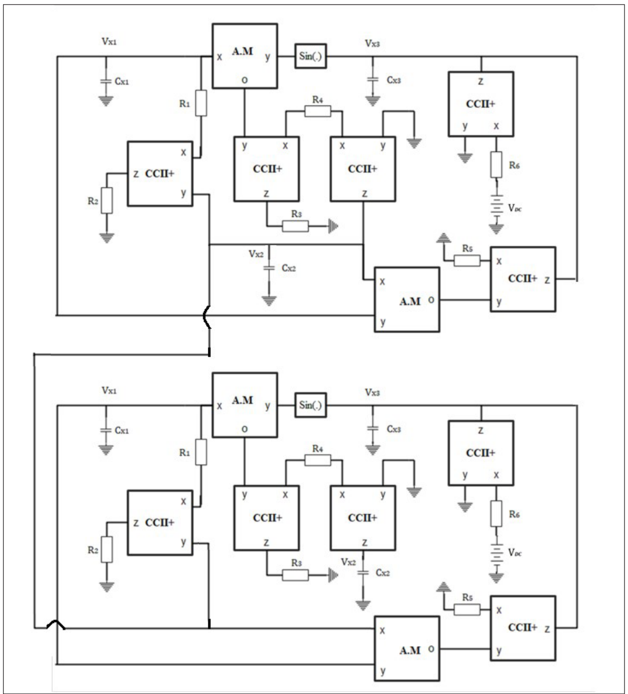

Figure 12. Synchronization of transmitter and receiver circuits by using signal transfer method

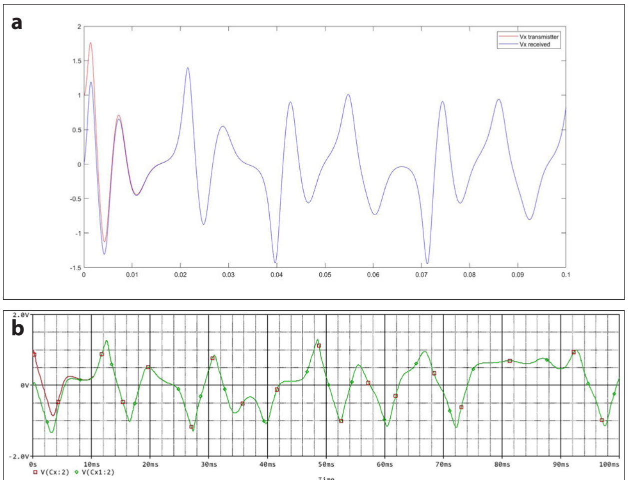

Figure 13. a, b. Transmitter and Receiver Vx in MATLAB for signal transfer method (a) Transmitter and Receiver Vz in PSpice OrCAD for signal transfer method (b)

$$\dot{x}_6 = \frac{V}{c_{x_3} * R_6} - \frac{k x_4 x_2}{c_{x_3} * R_5} \quad (7.3)$$

The block diagram of the method is given in Figure 12. The mathematical analysis of V_x (x_1 and x_4) signals drawn by MATLAB is given in Figure 13a. The simulation results obtained are given in Figure 13b. The system gets synchronized after approximately 15 ms in MATLAB and it takes approximately 10 ms in PSpice. With this method, the system is synchronized relatively faster. Besides, there is no need to use an extra active and passive element.

Table 2. NIST test results

Statistical test	p value	p value (KS)	Proportion
Frequency	0.066882	0.031899	1.0000
Block frequency M = 128	0.739918	0.364893	1.0000
Cumulative sums (forward)	0.534146	0.495070	1.0000
Cumulative sums (reverse)	0.213309	0.261194	0.9000
Runs	0.350485	0.107286	1.0000
Longest run	0.350485	0.493332	1.0000
Rank	0.213309	0.393105	1.0000
FFT	0.350485	0.928316	1.0000
Nonoverlapping template (min) M = 9	0.008879	0.015736	0.9000
Nonoverlapping template (max) M = 9	0.991468	0.853972	1.0000
Overlapping template M = 9	0.350485	0.682616	0.9000
Approximate entropy M = 10	0.739918	0.799079	0.9000
Serial(1) M = 16	0.350485	0.810288	1.0000
Serial(2) M = 16	0.350485	0.438950	1.0000
Linear complexity M = 500	0.911413	0.769873	1.0000

RNG

For the RNG, samples were obtained from the x_1 and x_3 time series produced by the chaotic generator, with the sampling frequency of 0.78 Hz. If the sample taken is a positive value, logic 1 information is recorded otherwise logic 0 information is recorded. A bit sequence is obtained by applying XOR operation to these logical values. The length of the resulting bitstream is 309 360. For NIST tests, 10 bitstreams were obtained. Used NIST parameters are taken for Block Frequency Test - block length as 128, Overlapping Template Test - block length as 9, Approximate Entropy Test - block length as 10, Serial Test - block length(m) as 16, and Linear Complexity Test - block length as 500. These arrays were NIST [39] tested for randomness control. The bitstreams have passed all the tests, available for the bitstreams of provided length. The NIST test results are given in Table 2. In this way, the resulting random bitstream can be used in image encryption methods that require random bits [46]. The RNGs obtained by solving the equations algorithmically are called pseudo random number generators (PRNG), whereas the generators that are based on hardware and can be affected by environmental factors are called true RNGs (TRNGs). It has been shown that a TRNG can be obtained by implementing the proposed circuit in hardware with proposed integrated circuits and passive circuit elements.

Conclusion

According to the schematic diagram of the circuit in the article in which the equation set is referenced [22], a structure containing fewer active elements is proposed in this study. However, the sine block used in the diagram is not a physically available active element. Therefore, in the reference study, physical realization has been made with a microcontroller by using digital realization instead of analog. However, there are studies in the literature that include current-mode elements and chaotic oscillator implementation and mention the advantages of current-mode operation [47]. In a similar study based on CCII+, an alternative to the Chua oscillator structure [48], which is widely used in the literature, is proposed. Although Chua diode structure can be realized with five active elements based on op amps [49], the nonlinear term can be produced with 2 CCII+ elements in the study. This study aims to emphasize the advantages of CCII +-based structures compared to the classical op amp-based structure. This advantage is to save space with the ability to create structures with fewer active and passive elements.

In this study, all mathematical analyses and simulation results are in good agreement. Successful chaotic synchronization has been achieved in MATLAB and PSpice simulations using two different synchronization methods. Bitstreams generated from XOR of two signals passed all the NIST tests. Thus, the system is feasible to use in chaotic encryption algorithms and secure communication.

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