

Selection of Input–Output Pairing and Control Structure Configuration Using Interaction Measures for DC–DC Dual Input Zeta-SEPIC Converter

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Cite this article as: P. Thota, A. R. Bhimavarapu and V V S Bhaskara Reddy Chintapalli, "Selection of input–output pairing and control structure configuration using interaction measures for DC–DC dual input zeta-SEPIC converter," *Electrica*, 23(1), 95-106, 2023.

ABSTRACT

Designing of automated devices for industrial and other applications is gaining momentum day by day, in particular with power electronic devices due to possible faster control. The efficiency of the devices depends on sensing the appropriate output signal and feeding the same to an appropriate input. In other words, selection/ pairing of appropriate input–output (IO) variable is very much essential for the effective operation of the device. In view of this, this study has focused on various procedures to identify the efficient IO pairing for designing a closed-loop system for Dual Input Zeta-SEPIC (DIZS) converter using interaction measures. To identify the best IO pairing for the selected DIZS converter, DC gain-based interaction measures and non-DC gain-based interaction measures are used. From the interaction measures using the methods listed earlier, selection of the best IO pairing, studies have been considered in this work. For the selected converter, to select the best IO pairing, studies have been considered for various cases: (1) with designed parameters; (2) with changes in the operating conditions; and (3) with changes in the values of the designed parameters. All the studies have been carried out using Matlab and PSIM simulation software.

Index Terms—Control structures, DC–DC converter, interactions, IO pairings, modeling, zeta-SEPIC converter

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E-mail: phanindras.thota@gmail.com Received: January 2, 2022

Revised: July 7, 2022

Accepted: July 19, 2022

Publication Date: November 12, 2022 DOI: 10.5152/electrica.2022.21184



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I. INTRODUCTION

Designing of automated applications for industries with power electronic converters with the combination of sophisticated controllers has been increasing day by day due to competitive environment between the industries. Designing of automated controller for particular converter is crucial for the efficient operation of converters and is becoming critical from the controller design point of view, as these automated applications are multivariable (input/output) in nature, and controlling is a challenging task due to the presence of interactions between the inputs and outputs. In multi input multi output (MIMO) systems, a particular output may affect the multiple inputs, or conversely, a particular input may be affected by the multiple outputs while operating with closed-loop configuration. If such systems are not analyzed properly, sometimes it may lead to low behavior/performance of the system [1] under closed-loop operating conditions. Often, inappropriate input-output (IO) pairing leads to closed-loop instability. Furthermore, sometimes, with a particular controller structure, the converter operates satisfactorily under steady-state operating conditions and may fail to exhibit desired performance under dynamic operating conditions like change of load, sudden disturbances, etc. [2]. In view of the above, selection of the best IO pairing and finest controller structure configuration (CSC) is essential to achieve an efficient outcome from the application. Interaction measures (IM)-based method is one of the commonly used technique to find the best IO pairing and robust CSC of MIMO systems. Methods used to assess IMs are broadly classified into two different categories viz.: 1) DC gain-based interaction measures (DCGIM) and 2) non-DC gain-based interaction measures (NDCGIM). In DCGIMs, pairing of the IO variables will be done using the DC gain matrix at 0 frequency, whereas in NDCGIMs, the IO variables paring will be done based on the interactions over a range of the frequencies, that is for a specific range of frequency (or) for entire frequency range (Gramian-based interaction measures). Generally, an appropriate CSC selection helps in minimizing the closedloop interactions between the controlled outputs and manipulated input variables. For finding the appropriate CSC based on IO pairing for any MIMO system transfer function matrix (TFM), the quantification of IMs and the nature of the application/plant play an important role. Therefore,

in order to achieve the desired output characteristics, selection of appropriate controlling input and corresponding controllable output pairing for a specific CSC is very much essential. In this work, IO pairing-based CSC selection using the interaction measures (IMs) of the selected Dual Input Zeta-SEPIC (DIZS) converter is presented. In Section II, a brief review of the existing literature on interactions is presented and in Section III, modeling and TFM of the selected converter are discussed. DC gain-based IMs and non-DC gain-based IMs are formulated with their pairing rules in Section IV. Quantification of IMs and selection of IO pairing-based CSC for DIZS converter for different cases are evaluated in Section V.

II. LITERATURE REVIEW

Interaction measures were introduced by Mitchell and Webb in 1960, where interaction quotient is also introduced for designing of decentralized control structure configurations [3]. In 1996, the relative gain array (RGA) method was introduced by Bristol for IO pairing selection of LTI systems only based on steady-state gains [4, 5]. In 1971, [6, 7] Niederlinski developed a useful tool named Niederlinski Index (NI), which is used to investigate the interactions and control loop pairings along with RGA pairing rules. If the NI value of the system is less than 0 (NI < 0), it indicates that the system is unstable. To have a stable system, the NI should be positive for the selected IO pairing [1, 8]. Relative gain array considers only the steady state gains at 0 frequency, that is, system dynamics are not taken into account. To overcome these deficiencies, frequency-based concepts like effective relative gain array (ERGA) proposed by Xiong et al. [9] in 2005 and effective relative energy array (EREA) introduced by Monshizadeh Naini et al. [10] in 2009, are used. Effective relative gain array and EREA use product of steady-state gain and bandwidths using transient and steady-state information for IO pairing. Applications of RGA and NI interactions are limited to decentralized controller selection and it is not adequate to attain the set of control targets for MIMO systems due to high degree of interactions. This can be overcome by using the Gramian-based interaction approach defined by Conley and Salgado in 2000 [11] and is a modified version of the investigations of khaki-Sedigh and Shahmansourian in 1996 [12]. In Gramian-based IO paring, interactions are measured by using observability Q and controllability P Gramian matrix calculated using Lyapunov equations. In Gramian-based IMs, whole frequency range is taken into account for a single measure. Some of the Gramianbased IMs investigated in [7, 13, 14] are based on participation matrix (PM) and were proposed by Conley and Salgado in 2000. H₂Norm IMs proposed by Birk and Medvedev in 2003 [15, 16] are also available for IMs assessment. In [17], Veerachary et al. designed a centralized controller for two-input multi-port DC-DC using RGA and ERGA IMs. Design of digital diagonal controller for two-input DC-DC converter is proposed by Reddy et al. in [18] by estimating the RGA and PM IMs. Reddy et al. in [19] designed a digital controller for two-input integrated DC-DC converter using Gramian-based IMs where RGA IM failed to suggest the CSC. Selection of control structure for a unique fifth-order DC-DC converter using RGA IM is presented by Vijay Kumar Tewari et al. [20] and given that de-centralized controller is suitable for closed-loop operations.

In the literature, most of the authors made use of any one of the IM techniques available in the literature to identify the suitable CSC. Some authors used any of the other available methods as a special case, when some of the commonly used methods fail to suggest a CSC for the selected application. Some IM techniques use DC components and some uses frequency components with different ranges. As different techniques use different components in suggesting the suitable CSC structure, sometimes, the selected CSC for the chosen application may fail to exhibit satisfactory behavior under steady state, under dynamic condition, or both.

In this study, to design a more accurate closed-loop system with suitable CSC for DIZS converter, different IM techniques with DC and frequency components are considered.

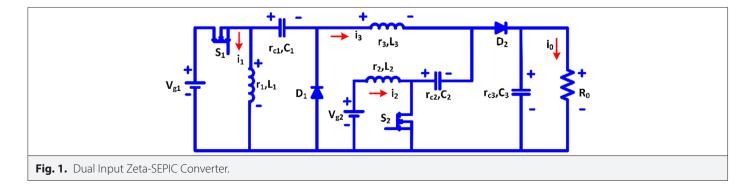
III. MODELING OF DC-DC CONVERTER

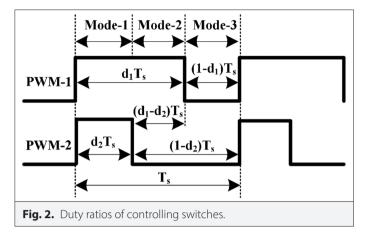
The DIZS type, non-isolated DC-DC converter [21] configuration shown in Fig. 1 has six energy storage elements (L₁, L₂, L₃, C₁, C₂, and C_3) with two controlling switches (S_1 and S_2).

The sequential ON and OFF switches regulate the duty ratios (d_1 and d_2 in order to drive the load in three different modes of operations (mode 1, mode 2, and mode 3) with two voltage sources V_{q1} and V_{q2} in different switching time intervals over a switching time period T, for $d_1 > d_2$ (Fig. 2).

The state space equations attained from mode 1, mode 2, and mode 3 operations of DIZS converter are described in (1), (2), and (3). In order to analyze DC-DC converter, the state space model of any one mode (mode 1, mode 2, and mode 3) represented in (1), (2), and (3) is not sufficient to extract the characteristics of the converter.

$$Y(t) = E_2 X(t) + F_2 U(t)$$





Therefore, state space average of all the three modes of operation for a switching period T(s) is formulated in (3) based on average conduction period of switches (S₁ and S₂) and is considered to derive the TFM G(s) using small-signal modeling analysis. The small-signal model of the converter is given in (6).

$$\overset{\bullet}{X} = A_{avg}X(t) + B_{avg}U(t)$$

$$Y(t) = E_{avg}X(t) + F_{avg}U(t)$$
(4)

where

$$A_{avg} = d_2 A_1 + (d_1 - d_2) A_2 + (1 - d_1) A_3$$

$$B_{avg} = d_2 B_1 + (d_1 - d_2) B_2 + (1 - d_1) B_3$$

$$E_{avg} = d_2 E_1 + (d_1 - d_2) E_2 + (1 - d_1) E_3$$

$$F_{avg} = d_2 F_1 + (d_1 - d_2) F_2 + (1 - d_1) F_3$$
(5)

$$\hat{x} = A\hat{x} + B\hat{u} + (\underbrace{(A_2 - A_3)X + (B_2 - B_3)U}_{\alpha_1})\hat{d}_1 + (\underbrace{(A_1 - A_2)X + (B_1 - B_2)U}_{\alpha_2})\hat{d}_2 \\ \hat{y} = E\hat{x} + F\hat{u} + (\underbrace{(E_2 - E_3)X + (F_2 - F_3)U}_{\beta_1})\hat{d}_1 + (\underbrace{(E_1 - E_2)X + (F_1 - F_2)U}_{\beta_2})\hat{d}_2$$
(6)

In DIZS converter, the two controllable output variables (V_0 and i_{g2}) are controlled by two input variables (d_1 and d_2). Therefore, the TFM of the DIZS converter is in the order of 2 × 2. The load voltage $V_0(s)$ is controlled by the two inputs (d_1 and d_2) and is represented independently in (7).

$$\frac{v_0(s)}{d_1(s)} = E^* (sI - A)^{-1} * \alpha_1 + \beta_1$$

$$\frac{v_0(s)}{d_2(s)} = E^* (sI - A)^{-1} * \alpha_2 + \beta_2$$
(7)

Similarly, the output current $i_{g2}(s)$ is controlled by the two inputs $(d_1 \text{ and } d_2)$ and is represented independently in (8).

$$\frac{i_{g_2}(s)}{d_1(s)} = P^* (sI - A)^{-1*} \alpha_1 + \beta_3$$

$$\frac{i_{g_2}(s)}{d_2(s)} = P^* (sI - A)^{-1*} \alpha_2 + \beta_4$$
where
$$\beta_3 = \left[(P_2 - P_3)X + (F_2 - F_3)U \right]$$
(8)

 $\beta_4 = \left\lceil (P_1 - P_2)X + (F_1 - F_2)U \right\rceil$

P is the row vector corresponding to output i_{a2}

 $P = P_1d_1 + P_2(d_1 - d_2) + P_3(1 - d_1)$. From (6) to (8), the TFM of the converter system is expressed in (9).

$$\begin{bmatrix} \hat{\mathbf{v}}_0(s) \\ \hat{\mathbf{j}}_{g2}(s) \end{bmatrix} = \begin{bmatrix} \mathbf{G}_{11}(s) & \mathbf{G}_{12}(s) \\ \mathbf{G}_{21}(s) & \mathbf{G}_{22}(s) \end{bmatrix} \begin{bmatrix} \hat{\mathbf{d}}_1(s) \\ \hat{\mathbf{d}}_2(s) \end{bmatrix}$$
(9)

where

$$G_{11}(s) = E^* [sI - A]^{-1} * \alpha_1 + \beta_1$$

$$G_{12}(s) = E^* [sI - A]^{-1} * \alpha_2 + \beta_2$$

$$G_{21}(s) = P^* [sI - A]^{-1} * \alpha_1 + \beta_3$$

$$G_{22}(s) = P^* [sI - A]^{-1} * \alpha_2 + \beta_3$$

 $\hat{v}_0(s)$ and $\hat{i}_{g2}(s)$ are the two outputs and $\hat{d}_1(s)$ and $\hat{d}_2(s)$ are the inputs of the converter.

IV. INTERACTION MEASURES

The IMs of any MIMO system provide the relationship between the input and output signals. In this study, DCGIM using RGA and NI and NDCGIM using Gramian-based IMs using PM and H₂Norm are considered. Effective relative gain array and Effective relative energy array IMs using specific frequency are also considered in this study. A brief introduction of the above-listed methods is presented in this section as follows:

A. DC Gain-Based Interaction Measures

In DCGIM, selection of the IO pairing of a system depends on steadystate gain matrix (a DC gain matrix). The DCGIM quantifies the steady-state interaction between the IO variables. The most common approach of DCGIM is RGA which determines the IO pairing based on the open-loop and closed-loop system properties. Using RGA, quantification of interactions for a MIMO system is evaluated from the DC gain transfer function matrix.

1) Relative Gain Array

Relative gain array used the non-singular DC gain matrix G (0) to assess the interactions among the available inputs and outputs. RGA of a (2×2) matrix is given in (10).

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} \lambda & (1-\lambda) \\ (1-\lambda) & \lambda \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$
(10)

where λ is the relative gain.

Based on the definition of RGA, it is observed that the diagonal elements (λ) and off-diagonal elements ($1 - \lambda$) are equal. The IO pairing rules in RGA-based approach are using the magnitude of the relative gain (λ) [2] and are furnished below.

Pairing Rules of RGA:

Rule-1 ($\lambda = 1$): If the value of $\lambda = 1$, there is no two-way interactions effect on IO pairing u₁-y₁ and u₂-y₂, that is, the impact of y₁ on u₂ and y₂ on u₁ does not exist. In practical scenarios, the value of λ close to 1 is highly preferable.

Rule-2 ($\lambda = 0$): If the value of $\lambda = 0$, the diagonal elements become 0 and the only possible IO pairing is off-diagonal pairing $u_1 - y_{2'} u_2 - y_1$.

Rule-3 ($0 < \lambda < 1$): If the value of the λ lies between 0 and 1, and all the system loops are closed, the selected IO pair gain of the system will be increased. The value of $\lambda = 0.5$ indicates worse IO pairing among the control loops.

Rule-4 ($\lambda < 0$): If the value of λ is negative, the change of sign in output leads to instability, so IO pairing with negative value of RGA should be avoided.

Rule-5 ($\lambda > 1$): If the value of the λ is greater than 1, IO pairing with large value of RGA indicates that the system is sensitive and ill-conditioned [1]. Larger value of λ forms a stronger interaction which is difficult to control.

The steady-state gain matrix (0) of 2×2 system is given in (3) and is used to compute the RGA.

$$G(0) = \begin{bmatrix} G_{11}(0) & G_{12}(0) \\ G_{21}(0) & G_{22}(0) \end{bmatrix}$$
(11)

The mathematical formulation of RGA is given in (12)

$$RGA(G) = G(0) \otimes G(0)^{-T}$$
(12)

where the operator \bigotimes is Hadamard product.

Relative gain array provides the best IO pairing considering 0 frequency, but it neglects the internal dynamics of the system over different frequencies. Hence, IO pairing using RGA may lead to inaccurate pairing; therefore, IO pairing using only steady-state gain is not satisfactory as control loops may interact dynamically. Niederlinski Index was defined [22] to overcome the above difficulties and is helpful in eliminating the unworkable IO pairings of a closed-loop system where RGA fails to compute. Relative gain array rules along with NI are extensively used in selecting the IO pairing. Niederlinski Index is defined as,

$$NI = \frac{\det(G(0))}{\det(\hat{G}(0))}$$
(13)

where G(0) matrix is formed by the diagonal elements with zeros in the off-diagonal. While using RGA along with NI for IO pairing, the following rules are used.

Pairing Rules of Relative Gain Array with Niederlinski Index:

- RGA element value should be positive and close to 1.
- NI value should be positive.
- Large values of RGA should be avoided while selecting the IO pairing.

B. Non-DC Gain-Based Interaction Measures

Non-DC gain-based interaction measure quantification overcomes most of the disadvantages of the IMs associated with conventional RGA. The quantification of these IMs is based on the energy and frequency of the transfer function elements.

1) Gramian-Based Interaction Measures

Gramian-based IMs are also known as energy-based interactions. In GBIM, whole dynamics of the system are considered for the quantification of the interactions by considering entire frequency range and can take only single measure. Gramians are matrices that define the observability and controllability properties of a linear system. Gramian matrices are formed for MIMO systems with various possible Single Input Single Output (SISO) sub-systems, each describing possible single input and single output combination of the given MIMO system. The observability Gramians (Q_i) are able to observe the system states from system outputs and the controllability Gramians (P_i) are able to control the system states from system inputs. Pairing of IO variables of the system is addressed using Q and P matrices. In this study, participation matrix and H₂Norm-based methods are considered to demonstrate the IO pairing selection procedure for the selected DIZS converter.

a) Participation Matrix: The quantification of interactions of DIZS converter using PM is addressed by evaluating the trace of the Gramian product $P_i * Q_j$, where P_i and Q_j are controllability and observability Gramians of the specific sub-system matrix (A, b_i, c_j, 0) of DIZS TFM G_{ij} (s) and these matrices have to satisfy the (14), (15), and (16).

$$P = \int_{0}^{\infty} e^{At} B B^{T} e^{At} dt$$
 (14)

$$Q = \int_{0}^{\infty} e^{At} C^{T} C e^{At} dt$$
(15)

where A is the system matrix, $b_{\rm i}$ is the ith column of the input matrix B, and $c_{\rm i}$ is the row matrix corresponding

$$\begin{bmatrix} \Phi \end{bmatrix}_{j} = \frac{tr(P_{j}Q_{i})}{tr(PQ)} = \frac{1}{tr(PQ)} \begin{bmatrix} tr(P_{1}Q_{1}) & tr(P_{2}Q_{1}) \\ tr(P_{1}Q_{2}) & tr(P_{2}Q_{2}) \end{bmatrix}$$
(16)

where

 $tr(PQ) = tr(P_1Q_1) + tr(P_2Q_1) + tr(P_1Q_2) + tr(P_2Q_2)$

b) H_2 Norm: The interaction quantification of H_2 Norm for IO pairing analysis of DIZS converter using TFM $G_{ii}(s)$ is expressed in (17)

$$\left[\Sigma_{2}\right]_{j} = \left[\frac{G_{ij}(s)}{\sum\limits_{k,j} G_{ij}(s)}\right]$$
(17)

 H_2 Norm can be interpreted as the energy of the impulse response. The H_2 Norm of a stable system $G_{ij}(s)$ with state space description of (A, B, C, 0) is evaluated as,

$$G_{ij}(s) = \sqrt{tr(B^{\mathsf{T}}QB)} = \sqrt{tr(CPC^{\mathsf{T}})}$$
(18)

 H_2 Norm interaction quantification can be measured using either controllability (P_i) or observability (Q_j). H_2 Norm as a measure of controllability is expressed in (19).

$$\begin{bmatrix} \Sigma_2 \end{bmatrix}_{j} = \begin{bmatrix} \frac{\sqrt{tr(CPC^{\top})}}{\sum\limits_{k,l} \sqrt{tr(CPC^{\top})}} \end{bmatrix}$$
(19)

2) Frequency-Based Interaction Measures

The quantification of interactions using FIMs for IO pairing can also be analyzed based on the information on steady-state gain and bandwidths of transfer function elements. In this study, ERGA and EREA are considered to demonstrate the IO pairing.

a) Effective Relative Gain Array: Effective relative gain array is an extension of the method using RGA and NI and uses steady-state gain and system bandwidth along with the procedure adopted in selecting the IO pairing using RGA and NI. Effective relative gain array is formed by characterizing the energy transmission ratio of the converter transfer function and provides information on both phase and gain changes when all other loops are closed. Effective relative gain array computation is based on effective gain matrix (*E*) and its expression is given in (20).

$$E = G(0) \otimes \Omega \tag{20}$$

where G(0) is steady-state gain matrix and Ω is bandwidth matrix. The Ω of 2 \times 2 system is

Band Width Matrix
$$\Omega = \begin{bmatrix} \omega_{c,11} & \omega_{c12} \\ \omega_{c,21} & \omega_{c22} \end{bmatrix}$$
 (21)

where $\omega c_i i j$; = critical frequency

Effective relative gain array is expressed as

$$\Lambda_{ERGA} = E \otimes E^{-T} \tag{22}$$

Pairing Rules of ERGA

- Select the ERGA elements values closest to 1.
- NI value should be positive.
- All paired ERGA elements should be positive.

b) Effective Relative Gain Array: In FIMs, EREA is one of the approaches in selecting the appropriate IO pairing. The IM analysis of EREA is similar to ERGA by considering the steady-state gains and bandwidths of the TFM *G*(s) elements. Effective Relative Gain Array is expressed in (23).

$$EREA = E \otimes E^{*^{-T}}$$
(23)

where

$$E^* = |G(0)| \times G(0) \times \Omega \tag{24}$$

The matrix E^* is the Hadamard product of absolute DG gain value, DC gain value, and bandwidth matrix of the TFM G(s). Absolute DC gain value |G(0)| is represented in (25).

$$|G(0)| = \begin{vmatrix} G_{11}(0) & G_{12}(0) \\ G_{21}(0) & G_{22}(0) \end{vmatrix}$$
(25)

Pairing rules of EREA:

- Select the EREA elements values closest to 1.
- NI value should be positive.
- All paired EREA elements should be positive.
- Avoid large EREA elements.

V. RESULTS

Dual Input Zeta-SEPIC converter design is particularly for low/ medium power applications. The designed converter parameters are listed in Table I and the corresponding output voltage of 32 V with a nominal power rating of 102 W.

TABLE I. PARAMETERS OF DIZS CONVERTER		
Parameter	Value	
Inductor L ₁ , L ₂ , L ₃	300 µH	
Capacitor C ₁ , C ₂	50 µF	
Capacitor C ₃	200 µF	
Voltage source V_{g_1}	40 V	
Voltage source V_{g^2}	12 V	
Resistor	10 Ω	
Output voltage V _o	32 V	
Duty ratio d,	0.42	
Duty ratio d ₂	0.2	
Switching frequency f _s	50 Khz	

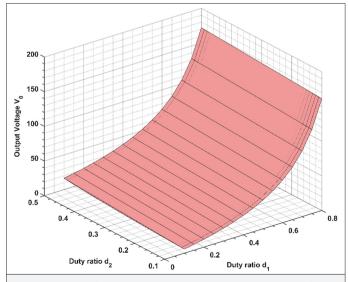


Fig. 3. D-plot of the DIZS converter with wide variations of duty ratios.

The power range of the converter may vary between 64 and 153 W, that is \pm 50% of rated power with the corresponding variations in the duty cycles. D-plot of the converter is shown in Fig. 3. In Fig. 3, to verify the converter range of outputs, duty cycles are varied in wide range and the corresponding output voltage is between 6 V and 172 V. However, the converter is designed for 32 V, and if a variation of \pm 10% is permitted, the operating output voltage range of the converter becomes 28.8–35.2V and the permitted range for duty cycles d_1 and d_2 becomes 0.4–0.45 and 0.16–0.2, respectively, to get the permitted output range.

This information is shown in Fig. 4. To identify the best IO pairing and suitable CSC for the selected DIZS converter, three cases are considered in this work viz.: 1) using designed parameters, 2) by considering variations in the operating conditions, that is variations in the sources and loads, and 3) by considering the uncertainty in the designed parameters within the range of $\pm 20\%$ variation.

A. Case-1: Converter with Designed Component Values

Computational procedure to select the IO paring using NDCGIMs and DCGIMs for the selected DIZS converter with designed component values is demonstrated in this case.

1) Computation of Relative Gain Array

DC gain matrix G(0) calculated using TFM G(s) of DIZS converter is required to apply the RGA-based rules to select the IO pairing and is given in (9). Elements of the TFM G(s) are given in (26).

$$G(s) = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix}$$
(26)

where

$$G_{11} = \frac{9800s^{5} + 1.067e^{9}s^{4} + 9.72e^{12}s^{3} + 1.022e^{17}s^{2} + 3.748e^{20}s + 1.822e^{24}}{s^{6} + 2696s^{5} + 9.145e^{7}s^{4} + 1.479e^{11}s^{3} + 2.31e^{15}s^{2} + 1.187e^{18}s + 1.153e^{22}}$$

$$G_{12} = \frac{2.149e^{9}s^{4} + 7.511e^{12}s^{3} + 1.202e^{17}s^{2} + 1.84e^{20}s + 1.179e^{24}}{s^{6} + 2696s^{5} + 9.145e^{7}s^{4} + 1.479e^{11}s^{3} + 2.31e^{15}s^{2} + 1.187e^{18}s + 1.153e^{22}}$$

$$G_{21} = \frac{-0.2113s^{6} + 1.767e^{4}s^{5} + 2.782e^{8}s^{4} - 60643e^{12}s^{3} + 5.627e^{15}s^{2} - 2.594e^{20}s - 3.656e^{22}}{s^{6} + 2696s^{5} + 9.145e^{7}s^{4} + 1.479e^{11}s^{3} + 2.31e^{15}s^{2} + 1.187e^{18}s + 1.153e^{22}}$$

$$G_{22} = \frac{4.959e^{4}s^{5} - 1.787e^{9}s^{4} - 2.215e^{12}s^{3} - 9.643e^{16}s^{2} - 1.216e^{20}s - 5.966e^{23}}{s^{6} + 2696s^{5} + 9.145e^{7}s^{4} + 1.479e^{11}s^{3} + 2.31e^{15}s^{2} + 1.187e^{18}s + 1.153e^{22}}$$

The DC gain matrix of the (26) is given in (27).

$$G(0) = \begin{bmatrix} 157.9524 & -3.1702 \\ 102.2347 & -51.7365 \end{bmatrix}$$
(27)

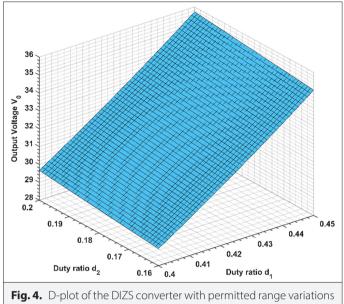
The inverse transpose of the DC gain matrix of the (27) is given in (28).

$$G(0)^{-T} = \begin{bmatrix} 0.0066 & 0.0130 \\ -0.0004 & -0.0201 \end{bmatrix}$$
(28)

Relative gain array of the DIZS converter is calculated using (12) and is given in (29).

$$RGA(G) = \begin{bmatrix} 1.0413 & -0.0413\\ -0.0413 & 1.0413 \end{bmatrix}$$
(29)

In this case, RGA is computed using the procedure defined in subsection "Relative Gain Array" of Section IV. From the RGA matrix, it



of duty ratios.

is observed that all the diagonal elements of RGA are greater than 1 and have the highest magnitude values when compared to off-diagonal elements. According to the RGA-based pairing rules, highest values of diagonal elements when compared to off-diagonal elements recommend diagonal IO pairing. Hence, RGA-based IO pairing procedure recommends the pairing of V_0 with d_1 and i_{g2} with d_2 for effective controlling.

2) Computation of Relative Gain Array with Niederlinski Index

Niederlinski Index is computed using (13), and for the selected converter, it is given by,

$$NI = \frac{\det(G(0))}{\det(\hat{G}(0))} = \frac{-7847.8}{-8171.9} = 0.9603$$
(30)

where G(0) and det($\hat{G}(0)$) are calculated using (11) and matrix formed by diagonal elements with zeros in the off-diagonal elements in G(0). According to the pairing rules of RGA, most preferable IO pairing of the converter is diagonal IO pairing as it was discussed in subsection "Computation of Relative Gain Array" of Section V and NI is supporting the same with positive value.

3) Computation of Participation Matrix

The PM described in (16) is computed using the controllability Gramian P and observability Gramian Q given in (14) and (15), respectively, from the TFM of the DIZS converter. In this method, TFM of the DIZS converter is assumed as Gramian matrix. In the Gramian matrix, each element is considered as a single sub-system and Gramian product P_i * Q_j is computed using the P and Q of the corresponding sub-system. Computation of PM is as follows:

For the sub-system G₁₁ corresponding to system variables V₀ and d₁, P₁ and Q₁ are computed. Using the P₁ and Q₁, Eigen values of the Gramian product of P₁ and Q₁ are computed and are 777 010, 638 660, 44 140, 45 520, 10, and 10. The trace of the Gramian product P₁Q₁ is equal to 1 505 350.

- Similarly, for the sub-systems G₁₂, G₂₁, and G₂₂, trace of the Gramian products is 253 730, 466 260, and 94 571, respectively.
- The trace of the DIZS converter is obtained by adding the traces of all sub-systems, G_{11} , G_{12} , G_{21} , and G_{22} and is equal to 2 319 911.

Participation matrix is obtained by normalizing the trace of each sub-system.

$$\begin{bmatrix} \Phi \end{bmatrix}_{j} = \frac{1}{2319911} \begin{bmatrix} 1505350 & 253730 \\ 466260 & 94571 \end{bmatrix}$$

$$= \begin{bmatrix} 0.6489 & 0.1094 \\ 0.2010 & 0.0408 \end{bmatrix}$$
(31)

To choose the IO pairing using PM, the following procedure is adopted:

- Select the element with highest magnitude in the PM.
- Discard all the elements in the row and column corresponding to element with highest magnitude except the element with highest magnitude.
- The elements which are remaining in the PM are chosen for IO pairing.

In this case, element PM (1,1) is having highest magnitude. Participation matrix (2,2) is the other surviving element after discarding the PM (1,2) and PM (2,1) as they are the elements corresponding to row and column of element with highest magnitude. Using the PM-based IO pairing procedure, pairing of V_0 with d_1 and i_{g2} with d_2 is recommended.

4) Computation of H2Norm

 H_2 Norm of a matrix can be calculated using the controllability Gramian matrix or Observability Gramian matrix. In this work, the computation of H_2 Norm for DIZS converter is carried out based on the controllability Gramian matrix as given in (19). Computation of H_2 Norm for each sub-system of the Gramian matrix is as follows:

- Consider the controllability Gramian of subsystem G_{11} and compute norm for G_{11} that is equal to 20 611.
- Similarly, computed the controllability Gramians of the remaining sub-systems, that is G₂₁, G₂₁, and G₂₂ are equal to 8340.7, 12 516, and 7380.2, respectively.
- The H₂Norm of the DIZS converter is obtained by adding the H2Norms of the individual subsystems and is equal to 48 848.
- To identify the elements for IO paring, normalized H₂Norm of the matrix is obtained by using the H₂Norms of the individual subsystems.

$$H_2Norm = \frac{1}{48848} \begin{bmatrix} 20611 & 8340.7 \\ 12516 & 7380.2 \end{bmatrix}$$

$$= \begin{bmatrix} 0.4219 & 0.1707 \\ 0.2562 & 0.1511 \end{bmatrix}$$
(32)

In this method, IO pairing is also done by adopting the same procedure used for PM. In this method, the only difference is that H₂Norm is used in place of PM. In this method, the results also suggest that IO pairing of V_0 with d_1 and i_{g2} with d_2 of the DIZS converter is most preferred in designing the controllers.

5) Computation of Effective Relative Gain Array

For the selected DIZS converter, the ERGA is computed using (22). In (22), E is computed using (20). Where Ω is bandwidth matrix obtained from the TFM *G*(s). To get the Ω matrix, the bandwidth values of each element of the *G*(s) are calculated and arranged in a matrix form by placing the bandwidth values in the corresponding locations of *G*(s). For the selected converter, bandwidth values of each sub-system are as given as follows:

Band Width of
$$G_{11} = 04923$$

Band Width of $G_{12} = 14799$
Band Width of $G_{21} = 03423$
Band Width of $G_{22} = 09592$

Band Width Matrix (
$$\Omega$$
) = $\begin{bmatrix} 4923 & 14799 \\ 3423 & 9592 \end{bmatrix}$ (34)

Using the G(0) and Ω , E is calculated using (20) and is given in (35)

$$E = \begin{bmatrix} 157.9524 & -3.1702 \\ 102.2347 & -51.7365 \end{bmatrix} \otimes \begin{bmatrix} 4923 & 14799 \\ 3423 & 9592 \end{bmatrix}$$

$$= \begin{bmatrix} 7.7760 & -4.6910 \\ 3.4997 & -4.9628 \end{bmatrix}$$
(35)

After getting *E*, *ERGA* is calculated using (22) and is given in (36).

$$ERGA = \begin{bmatrix} 7.7760 & -4.6910 \\ 3.4997 & -4.9628 \end{bmatrix} \otimes \begin{bmatrix} 0.1343 & 0.0947 \\ -0.0127 & -0.2105 \end{bmatrix}$$

$$= \begin{bmatrix} 1.0444 & -0.0444 \\ -0.0444 & 1.0444 \end{bmatrix}$$
(36)

In ERGA-based method, same procedure is adopted as it was done for RGA-based method for IO pairing. The difference between RGA and ERGA is in RGA where only DC gain matrix is used, whereas, in ERGA, bandwidths are also considered along with DC gain matrix.

6) Computation of Effective Relative Energy Array

In this method, EREA is computed using (23). The procedure adopted in this method is similar to ERGA. In this procedure, E^* is computed using (24) with absolute DC gain and DC gain calculated using (11) and bandwidths calculated using (21) of the DIZS converter transfer function matrix G(s). For the selected DIZS converter, absolute DC gain, DC gain, and bandwidths and E^* are given as follows:

$$G(0) = Absolute DC_{gain}$$

$$= \begin{bmatrix} 157.9524 & 3.1702 \\ 102.2347 & 51.7365 \end{bmatrix}$$
(37)

$$G(0) = \begin{bmatrix} 157.9524 & -3.1702 \\ 102.2347 & -51.7365 \end{bmatrix}$$
(38)

Band Width Matrix(
$$\Omega$$
) = $\begin{bmatrix} 4923 & 14799 \\ 3423 & 9592 \end{bmatrix}$ (39)

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IMs/Cases	Case-A		Case-B		Case-C	
RGA		0.1340 .1340	0.9312 0.0688	0.0688 0.9312	1.0408 -0.0408	-0.0408 1.0408
RGA with NI	0.8819		1.0739		0.9608	
PM		1069 0449	0.6414 0.2385	0.1162 0.1299	0.6608 0.1925	0.1091 0.0376
H ₂ Norm		1707 1511	0.4510 0.2511	0.1745 0.1595	0.4270 0.2549	0.1719 0.1462
ERGA		0.0109 .0109	0.9321 0.0679	0.0679 0.9321	1.0436 0.0436	-0.0436 1.0436
EREA		0.0109 .0109	0.9946 0.0054	0.0054 0.9946	[1.0016 0.0016	-0.0016 1.0016

	BLE II. SUMMARY OF THE RESULTS FOR DIFFERENT OPERATING CONDITIONS
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$E^* = \begin{bmatrix} 157.9524 & 3.1702 \\ 102.2347 & 51.7365 \end{bmatrix} \otimes$	
$\begin{bmatrix} 157.9524 & -3.1702 \\ 102.2347 & -51.7365 \end{bmatrix} \otimes \begin{bmatrix} 4923 \\ 3423 \end{bmatrix}$	14799 9592]
$E^* = \begin{bmatrix} 12.282 & -0.015 \\ 3.578 & -2.568 \end{bmatrix}$	

EREA is computed using (23) and is given in (41).

$$EREA = \begin{bmatrix} 12.282 & -0.015 \\ 3.578 & -2.568 \end{bmatrix} \otimes \begin{bmatrix} 0.0816 & 0.1136 \\ -0.0005 & -0.3901 \end{bmatrix}$$

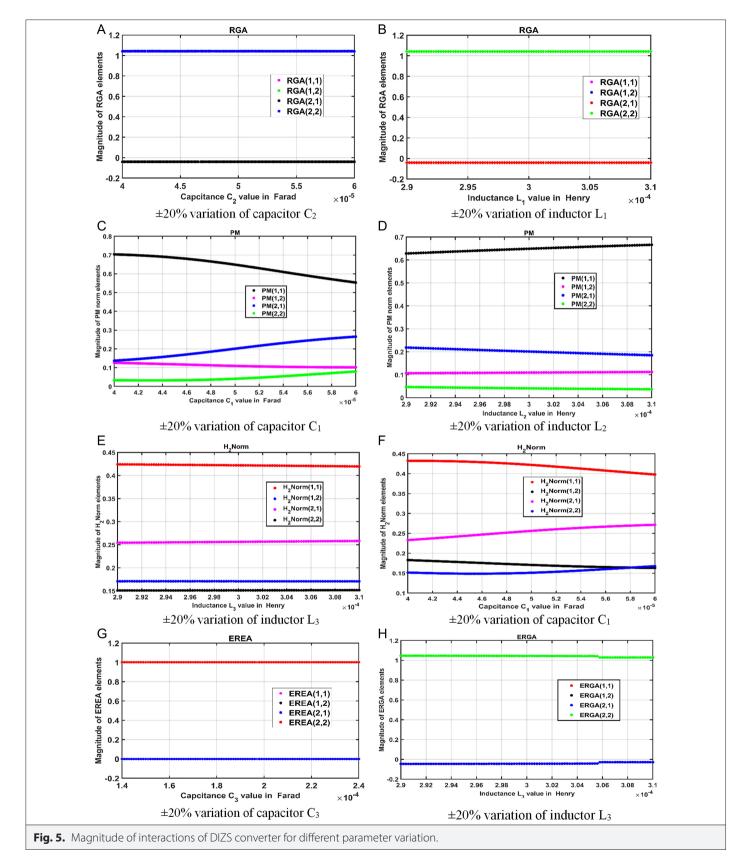
$$= \begin{bmatrix} 1.0017 & -0.0017 \\ -0.0017 & 1.0017 \end{bmatrix}$$
(41)

(40)

Effective relative energy array for nominal conditions also suggests the diagonal IO paring with high values in diagonal elements

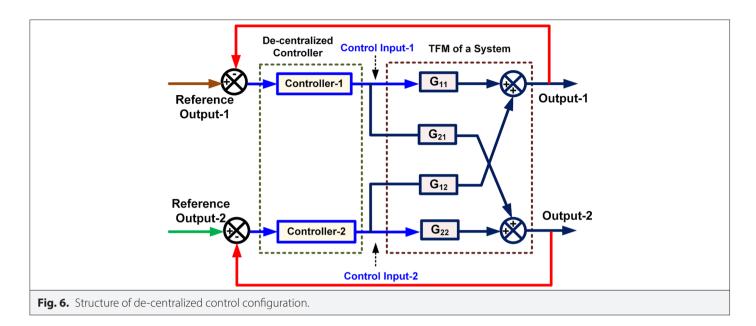
IMs/Cases	Case-X	Cas	Case-Y		Case-Z	
RGA	1.0413 -0.04 -0.0413 1.041		-0.0413 1.0413	[1.0413 0.0413	-0.0413 1.0413	
RGA with NI	0.9603	0.9	603	0.9	603	
PM	0.5393 0.0928 0.2900 0.0778		0.1212 0.0290	0.6694 0.1809	0.1150 0.0347	
H ₂ Norm	0.3970 0.155 0.2799 0.174		0.1899 0.1366	0.4306 0.2536	0.1765 0.1393	
ERGA	1.0283 -0.024 -0.0283 1.028		-0.0444 1.0444	[1.0536 0.0536	-0.0536 1.0536	
EREA	[1.0011 -0.00 0.0011 1.001		-0.0017 1.0017	1.0020 -0.0020	-0.0020 1.0020	

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compared to off-diagonal elements, that is EREA (1,1) is 1.0017 and EREA (2,2) is 1.0017. All the methods demonstrated, in this case, suggest diagonal IO pairing as best choice for designing the closed-loop systems for the selected DIZS converter.

B. Case-2: Converter with Different Operating Conditions In this case, computation of IMs at different operating points is chosen to study the impact of IO pairing on the converter operation. Studies have been carried out with variations in source voltages and



load resistance in the range of \pm 50%. Among the various cases considered, due to various constraints, only following three cases are presented in this section:

- Case-A: $V_{a1} = 48 \text{ V}$, $V_{a2} = 9.60 \text{ V}$, $R = 8 \Omega$.
- Case-B: $V_{q1} = 32 \text{ V}, V_{q2} = 14.40 \text{ V}, \text{ R} = 10 \Omega.$
- Case-C: $V_{q1} = 48 \text{ V}$, $V_{q2} = 14.40 \text{ V}$, $R = 12 \Omega$.

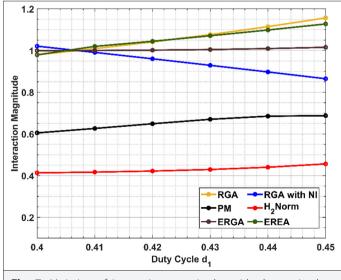
The IMs are computed for the three cases listed above using all the six methods demonstrated in the subsection "Converter with designed component values" of Section V and the results are presented in Table II. All the results presented in Table II show that diagonal IO pairing is the best choice for designing the closed-loop systems for selected DIZS converter.

C. Case-3: Converter with Changes in Different Parameter Values In this case, IMs for DIZS converter are computed with variations in parameter values of the energy storage elements, and L_1 , L_2 , L_3 , C_1 , C_2 , C_3 are computed with ±20% range of variations from its designed values. Among the many cases simulated, three random cases listed below are presented in this section.

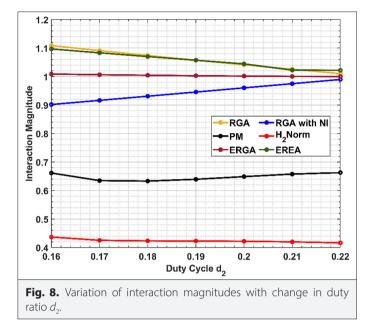
- Case-X: $L_1 = 270 \ \mu$ H, $L_2 = 300 \ \mu$ H, $L_3 = 270 \ \mu$ H, $C_1 = 45 \ \mu$ F, $C_2 = 55 \ \mu$ F, $C_3 = 200 \ \mu$ F
- Case-Y: $L_1 = 330 \ \mu\text{H}$, $L_2 = 330 \ \mu\text{H}$, $L_3 = 270 \ \mu\text{H}$, $C_1 = 45 \ \mu\text{F}$, $C_2 = 55 \ \mu\text{F}$, $C_3 = 200 \ \mu\text{F}$
- Case-Z: $L_1 = 300 \ \mu$ H, $L_2 = 270 \ \mu$ H, $L_3 = 270 \ \mu$ H, $C_1 = 45 \ \mu$ F, $C_2 = 45 \ \mu$ F, $C_3 = 180 \ \mu$ F

The IMs are computed for the three cases presented in this section using all the six methods demonstrated in subsection "Converter with designed component values" of Section V, and the results are presented in Table III. Observing all the results presented in Table III, it appears that the selection of diagonal IO pairing is more beneficial if a closed loop system is considered for the selected DIZS converter. From the results presented in subsections "Case-1: Converter with Designed Components", "Case-2: Converter with Different Operating Conditions", and "Case-3: Converter with Changes in Different Parameter values" of Section V, it is identified that for the selected DIZS converter, implementation of decentralized controller by selecting the diagonal IO pairing for efficient operation of closed-loop system. The magnitudes of computed IMs graphically presented for different parameter variations over a range of $\pm 20\%$ of its designed values are shown in Fig. 5a-h. General structure of the decentralized controller structure configuration recommended for selected DIZS converter with 2-inputs and 2-outputs is shown in Fig. 6.

In addition to the above, variation of the various indices used to select the IO pairing and suitable CSC within the permitted range of operation of the converter with duty cycles d_1 in the range of 0.4–0.45 and d_2 in the range of 0.16–0.2 is shown in Fig. 7 and 8, respectively. From Fig. 7 and 8, it is identified that, in the entire range of the operation, results of all the techniques considered in this work are pointing to the diagonal IO pairing and decentralized CSC.







VI. CONCLUSION

Due to the feature of faster control, power electronic-based devices are extensively designed and developed for small-scale to largescale applications. During the design process, selection of appropriate signal for closed-loop control plays a vital role in the performance of the system. Identifying the appropriate pair of output parameter and input parameter which has better interaction is crucial in the design of closed-loop systems. In this study, selection/pairing of appropriate IO variables for designing closed-loop system for DIZS converter using IMs is considered. Studies have been carried out to quantify the interactions between various IO pairs using 1) DCGIM like RGA, RGA with NI and 2) NDCGIM like PM, H₂Norm, ERGA, and EREA. With the designed parameters with the designed source voltages and loads, RGA computed is [1.0413 -0.0413; -0.0413 1.0413], NI computed is +0.9603, PM computed is [0.6489 0.1094;0.2019 0.0408], H₂Norm computed is [0.4219 0.1707;0.2562 0.1511], ERGA computed is [1.0444 -0.0444; -0.0444 1.0444] and EREA computed is [1.0017 -0.0017; -0.0017 1.0017]. Results obtained from all these methods suggest the diagonal IO pairing, that is, pairing of V_0 with d_1 and i_{a2} with d_2 , and decentralized CSC. Numerous studies have also been carried out using the modeling developed [21] for DIZS converter with 1) changes in the operating conditions and 2) changes in the values of the designed parameters. From the studies, it was identified that diagonal IO pairing and decentralized CSC are very much recommended in designing the closed-loop system for the DIZS converter. The results are tabulated for selected cases and are presented in Tables II and III.

Peer-review: Externally peer-reviewed.

Author Contributions: Concept – A.R.B., V.V.S.B.R.C., P.T.; Design – A.R.B., P.T.; Supervision – A.R.B., V.V.S.B.R.C.; Analysis and/or Interpretation – P.T., A.R.B., V.V.S.B.R.C.; Literature Review – P.T., A.R.B.; Writing – V.V.S.B.R.C., P.T.; Critical Review – V.V.S.B.R.C., P.T.

Declaration of Interests: The authors declare that they have no competing interest.

Funding: The authors declared that this study has received no financial support.

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