

# Formal Concept Analysis and Its Validity in Power Reliability Engineering

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## ABSTRACT

Sub-discipline of systems engineering that focuses on the capacity of equipment to operate without failure is reliability engineering. The capacity of a system or component to perform as expected under defined circumstances over an extended length of time is referred to as reliability. The approach of data analysis known as formal analysis of concepts is used to investigate the link between a collection of objects and their associated qualities (input termed as the formal context). Formal analysis of concepts is not only capable of identifying data groupings (concepts) and visualizing them but it can also extract rules that indicate the essential nature of the investigated environment. When it comes to reliability engineering, formal analysis of concepts may be a valuable tool. New theories in formal analysis of concepts in this domain are supported by mathematical proofs in this article. Commentary on formal analysis of concepts latest findings is also included prominently.

**Index Terms**—Electric power system, formal concept analysis, knowledge representation and reasoning, lattice structure, ontology, reliability engineering

## I. INTRODUCTION

In reliability engineering [1], the link between reliability evaluation and improvement is critical. Reliability engineering's purpose is to assess the product or process's inherent reliability and identify opportunities for improvement in that reliability [2-5]. Another objective of reliability engineering is to identify the most probable failures and then determine the best ways to prevent or minimize the impact of such failures. It is possible to conduct a variety of reliability evaluations when evaluating a product or process for dependability. Depending on the stage of the product's lifetime, several sorts of analysis are required. During the reliability phase; it is possible to find out what the outcome is, by looking at the reliability effects of design changes and corrections based on the analysis. As a result of this, it is feasible to identify potential issues with the products and systems that are being studied by conducting a variety of dependability assessments [6].

Power system reliability assessment is a critical component in the prediction organizing, blueprint design, and working principle of power systems [7-11]. The components of an electric power system are integrated in some planned and meaningful way. The goal of a reliability assessment is to identify appropriate metrics, criteria, and indices of reliability and dependability based on configuration import data. Generator units and system configuration, which relate to the individual units operating to satisfy the current or future demand, are key components for measuring produced dependability. Reliability indexes are probabilistic evaluations of a certain generating configuration's ability to satisfy the demand for electricity. Rather than being absolute measurements of system dependability, these indices are best understood as assessments of system-wide generation adequacy. The indices may be used to compare the relative dependability of various generating setups since they are responsive to fundamental characteristics like unit size and availability. It is considered a successful system if there is sufficient generating capacity (enough reserve) to meet peak demand (maximum demand). The risk of supply shortages in the system is first calculated by converging (mutually combining) the generating and load models. This arrangement is evaluated using probabilistic estimations of shortfall risk, which are utilized as indicators of bulk power system dependability [12].

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Formal concept analysis (FCA) [13] is a method for analyzing ideas, as the name suggests. Information science uses FCA to derive an ontology or concept hierarchy from a collection of objects and their attributes in a logical manner. All of the items in the hierarchy share certain attributes, and each sub-concept of that hierarchy represents a subset of the objects (and properties) in the ideas above it [35-37].

The primary goal is to demonstrate how FCA may be used to reliability engineering concerns in power systems, as well as its basic principles specifically in analyzing the cut-sets in a two-state system which is a well-known method for determining how it may go wrong. Formal concept analysis may directly answer reader-posted queries like: Which components are part of a certain state or cut-set? given an input table describing the link between component status (functioning or failed) and system status (failed). Who decides what constitutes a “minimum cut-set?” Cut-sets have a similar connection. Is it possible to have the same components in many cut sets? Along with multiple unresolved issues in FCA are discussed in this work, some of which have adequate evidence.

The study dwells on the literature review in Section II, the basic terminologies and theorem behind formal concept analysis is explored in Section III, the application of FCA in electric power system is covered in Section IV, and finally, the concluding remarks are placed in Section V. Let's have a look at some of the literature first before diving into the principles.

## II. LITERATURE SURVEY

Ontology has been playing its part in reliability engineering with specificity to power systems [39-43]. An ontology based on electrical power quality was constructed using data from transmission and distribution systems, such as frequency, flicker, and harmonics in [14]. There are many linguistic qualities that may be applied to any of the ontology's concepts in the proposed ontology, which helps it to enable linguistic applications. Power quality data generated by a countrywide power quality measuring system may be queried in a flexible manner thanks to the ontology's multilingual natural language interface. The ontology's contribution to a real-world application is shown via the implementation details of the interface and a few query samples.

The scientific community is being asked to explain the link between resilience, risk, and safety in the context of ethics in [15]. Depending on the response, the amount of risk-taking required to demonstrate resilience in complex socio-technical systems would be affected. To avoid the “zero injury” or “zero accident” fallacy, it is essential to remember that the concept of safety should always be evaluated in conjunction with an acceptable degree of risk. In the absence of an agreed-upon risk level, subjects of resilience must agree and maintain an appropriate level of risk regardless of whether the acceptable level is pre-defined in terms of workload, quality, and protection. “Resilience” is an example of a new category of safety that has the power to help society understand and accept political decisions to run particular technology in certain contexts [44].

Reliability specialists will benefit from the ontology-based method's reliability ontology and accompanying computer-aided tools in [16]. The first step was to define the ideas of dependability. Dependability ontology's aims, methodologies, and principles were all explored in detail, as were several research pertaining to the field of ontology engineering. In addition, the reliability ontology was built largely

with regard to the principles and attributes of reliability-aware software design in mind. The ontology-based reliability design tool's development experiences were also discussed.

As an e-board product, the goal of this work [17] is to improve dependability design. This requires a shared knowledge of reliability design information and its related ideas. Then we see various reliability design data. Product dependability information will be expressed using failure ontology and a related ontology. The Board-Level Electronic Product (BLEP) failure ontology framework provides ideas and linkages between reliability design principles. The BLEP failure hypostasis is explained in a hierarchical failure ontology framework. The skeleton approach is used to build the failure ontology model and convey it. Finally, an example BLEP illustrates the method's utility.

When designing an industrial process, dependability analysis is necessary. In the field of industrial automation, an application for autonomous fault tree creation based on ontology is shown [18]. A water tank was used to test the program. High abstraction is made possible by using this method. Because of this, designers may test a variety of settings and discover the most reliable one with no room for human mistake. In the future, the method is to be tested in real-world huge systems to see how well it performs.

In an eco-industrial park (EIP), a domain ontology for power systems was built and utilized in [19]. Data, information, and models were brought together in an EIP via the use of a web-based software platform. Ontologies developed by the JPS project are utilized by a variety of agents. These two agents and their execution architecture are explored in the study. Cross-domain interoperability has been improved by using web agents in case studies. Hermit, the reasoning engine that powers JPS, ensures that the knowledge graph is consistent. Automated processes and the capacity to transmit data over the Internet are among the benefits. It is concluded that it is possible to create smart factories that need little human interaction if the proposed technology is used to its maximum potential. In order for this to function, we would need Industry 4.0 technologies like semantic web ontology. More contextual information is believed to be added to the ontology, and validation criteria will be tightened in the first phase. This includes the building of ontologies for particular electrical engineering domains as part of the process. Standard description logic syntax will make this phase easier since it will enable the ontology to be realized in suitable languages, making this stage less burdensome [45, 46]. The focus of the next section will be on rule-based ontology-based systems that can make choices without the involvement of external actors.

Electric power knowledge theory is proposed in this study [20] to solve the problem of normalized modeled electric power knowledge for the management and analysis of electric power huge datasets. Current modeling approaches are deemed inadequate due to the system's high degree of interdependence and variety. New knowledge modeling techniques are provided through the use of semantic web technologies in electric power systems and other sectors. A whole new knowledge model is presented here, including its structure and constituents, as well as its basic computations and multidimensional reasoning technique. A simulation demonstrates the requirements of the electric power system operating standard. Electric power system standard modeling, multi-type data management, and unstructured data searching are only some of the ways the model and accompanying technologies are presented. According to the study, a powerful new model established here is capable of

adapting to a variety of knowledge representation demands for electric power data. In order to develop and broaden the application of the knowledge model in several fields, we may expect to see an increase in the use of electric big data technology in the future [47]. The concept of fuzzy logic in the field of FCA is explored in [43-49] where the studies not only discuss the uses of traditional fuzzy logic but also bipolar and m-polar contexts as input for the concept creation. Now we discuss in detail the fundamentals of FCA in the next section.

### III. FORMAL CONCEPT ANALYSIS

Formal concept analysis is termed as a method for analysis and representation of data in which the connection between a group of items and a certain set of qualities may be studied, which can reveal their structure. In 1981, Rudolf Wille introduced FCA [13], a subdivision of lattice theory [21]. Objects and attributes are represented via a cross table in FCA's first step. FCA generates: 1) groups that reflect "natural" notions in terms of the data's qualities and 2) a collection of implications that defines a particular dependence that occurs in the data. As an unsupervised machine learning methodology, formal concept analysis is a mathematical theory of idea hierarchies that draws on order theory. It is often employed as a way of knowledge representation. Hasse diagrams or partial order structures may be used to arrange ideas in a binary relation, such as a binary matrix that contains a collection of objects and their attributes (rows and columns). There are two sets of objects and characteristics for each notion in the final diagram, and these sets are completely encapsulated inside one another. Table I gives a reference to the symbols used in this article.

Definition 1: The set of objects or items is represented by " $G$ ." The corresponding attributes or properties are represented by " $M$ ." " $I$ " is a mapping function which says which objects correspond to which property. Specifically,

$$I \subseteq G \times M \quad (1)$$

$(a, b) \in I$  translates that " $a$ " has property " $b$ ."

This data matrix must include columns and rows with characteristics as required by the FCA. The entries in the matrix show how items and their attributes are linked. If a characteristic is present or absent, the number one or zero (one or zero) signifies that in most cases. The zeros are usually omitted when " $X$ " substitutes " $1$ ;" however, this is not always possible. As an additional benefit, this replacement ensures that the data matrix components cannot be misinterpreted as numbers.

Definition 2: The input to the knowledge representation is called as the formal context. Mathematically, it is a triple  $(G, M, I)$ .

As a representation, Table II shows an example of a context.

As seen in Table II, we can say that object " $a_1$ ," possess the properties " $\{b_1, b_2, b_4, b_5\}$ ." There is another distinct way to think about a formal context: in graph theory, the bigraph is defined as the set of vertices " $G$ " and " $M$ " that make up an object and the set of edges " $I$ " that connect these two sets (each edge is adjacent to one object and one attribute vertex). As a consequence, the findings of FCA may also be used in network analysis. The graph's incidence matrix serves as the formal environment in this instance.

**TABLE I.** NOTATIONS USED IN THE ARTICLE

Notation	Interpretation
$G$	Set of objects or items
$M$	Set of attributes or properties
$I$	Mapping function
$\alpha'$	Derivative function on a set $\alpha \subseteq G$
$\beta'$	Derivative function on a set $\beta \subseteq M$
$(G, M, I)$	Formal context
$(\alpha, \beta)$	Formal concept
$\Delta(\alpha, \beta)$	Set of formal concepts
$<$	Super concept
$\ll$	Binary relationship for Poset
$\eta$	Stability index
$GR$	Stability reduced graph
$N$	Set of vertices
$D$	Set of edges
$CS$	Cut-set

Definition 3: A derivative function produces the Galois connection for  $\alpha \subseteq G$  and  $\subseteq M$  :

$$\alpha' = \{b \in M \mid (a, b) \in I, \forall a \in \alpha\} \quad (2)$$

**TABLE II.** FORMAL CONTEXT

	$b_1$	$b_2$	$b_3$	$b_4$	$b_5$	$b_6$
$a_1$	X	X		X	X	
$a_2$			X	X	X	X
$a_3$	X		X		X	X
$a_4$	X		X		X	
$a_5$	X	X	X		X	
$a_6$		X		X		X
$a_7$	X	X	X	X		
$a_8$				X	X	X
$a_9$	X	X	X			
$a_{10}$			X	X	X	

(the set of attributes common to the object set in  $\alpha$ )

$$\beta' = \{a \in G \mid (a, b) \in I, \forall b \in \beta\} \quad (3)$$

(the set of objects common to the attribute set in  $\beta$ )

In Table II, if  $\alpha = \{a_1, a_2\}$ , then  $\alpha' = \{b_4, b_5\}$ ; which means that these are the set of properties which are common to the object set. Other relationships can also be gathered like:

$$\begin{aligned} a_1 &\subseteq a_2 \Rightarrow a'_2 \subseteq a'_1 \\ b_1 &\subseteq b_2 \Rightarrow b'_2 \subseteq b'_1 \end{aligned} \quad (4) \text{ and } (5)$$

$$a_i \subseteq a_i'' \forall a_i \subseteq Gs \quad (6)$$

$$b_i \subseteq b_i'' \forall b_i \subseteq M \quad (7)$$

$$a'_i = a_i''' \forall a_i \subseteq G \quad (8)$$

$$b'_i = b_i''' \forall b_i \subseteq M \quad (9)$$

Definition 4:  $(\alpha, \beta)$  is termed as a formal concept of the formal context  $(G, M, I)$  where  $\alpha \subseteq G$  and  $\beta \subseteq M$ , where  $\alpha' = \beta$  and  $\beta' = \alpha$ . Formal concepts are composed of two sets,  $\alpha$  (extent) and  $\beta$  (concept intent). All the objects in  $\alpha$  have a common attribute set  $\beta$ , which is why they are called sets.

Algorithm 1 deals with the generation of the set formal concepts  $\Delta(A, B)$  from the input context  $(G, M, I)$ . Table III reflects the formal concepts generated.

**Algorithm 1** Formal concept set generation

Generate formal concepts from the formal context  $(G, M, I)$ .

Input: Formal Context  $(G, M, I)$

Output: Set of formal concepts  $\Delta(A, B)$

```

1: procedure GENERATE— $\Delta(X, Y)$ 
2:    $\Delta = \phi, a_i = \phi$ 
3:   for each  $a_i \in G$  do
4:     if  $((a'_i = b_j) \ \&\& \ (b'_j = a_i))$ 
5:       Update  $\Delta(A, B) \leftarrow (a_i, b_j)$ 
6:       Delete for duplicate entries in  $\Delta(A, B)$ 
7:   end for
8: end procedure
```

As at this stage, it is indeed possible to establish a formal link between concepts. A natural order for formal ideas is sub concept–super concept, which is based on the inclusion connection between objects and characteristics. Here are the formal definitions of the sub concept–super concept relationship.

Definition 5:  $(a_i, b_i)$  is said to be the super concept of  $(a_j, b_j)$  if  $(a_i > a_j) \ \&\& \ (b_i < b_j)$ .

We generated a lattice as shown in algorithm 2 with the same sub concept–super concept relationship. Every node (or vertex) in this single-source, single-sink, labeled, directed acyclic graph (DAG) reflects a formal notion, while the edges indicate the conceptual ordering. If a direct route exists between the vertices of a DAG, two ideas may be compared; otherwise, they cannot be compared.

The generated lattice for the concepts is shown in Fig. 1.

**TABLE III.** GENERATED CONCEPTS

Concept <sub>id</sub>	A	B
C <sub>1</sub>	$\{\phi\}$	$\{b_1, b_2, b_3, b_4, b_5, b_6\}$
C <sub>2</sub>	$\{a_2\}$	$\{b_3, b_4, b_5, b_6\}$
C <sub>3</sub>	$\{a_6\}$	$\{b_2, b_4, b_6\}$
C <sub>4</sub>	$\{a_3\}$	$\{b_1, b_3, b_5, b_6\}$
C <sub>5</sub>	$\{a_2, a_3\}$	$\{b_3, b_5, b_6\}$
C <sub>6</sub>	$\{a_2, a_6\}$	$\{b_4, b_6\}$
C <sub>7</sub>	$\{a_1, a_7\}$	$\{b_1, b_2, b_4\}$
C <sub>8</sub>	$\{a_7\}$	$\{b_1, b_2, b_3, b_4\}$
C <sub>9</sub>	$\{a_1\}$	$\{b_1, b_2, b_4, b_5\}$
C <sub>10</sub>	$\{a_2, a_7\}$	$\{b_3, b_4\}$
C <sub>11</sub>	$\{a_1, a_2\}$	$\{b_4, b_5\}$
C <sub>12</sub>	$\{a_1, a_6, a_7\}$	$\{b_2, b_4\}$
C <sub>13</sub>	$\{a_5\}$	$\{b_1, b_2, b_3, b_5\}$
C <sub>14</sub>	$\{a_1, a_5, a_7\}$	$\{b_1, b_2\}$
C <sub>15</sub>	$\{a_5, a_7\}$	$\{b_1, b_2, b_3\}$
C <sub>16</sub>	$\{a_1, a_5\}$	$\{b_1, b_2, b_5\}$
C <sub>17</sub>	$\{a_3, a_4, a_5\}$	$\{b_1, b_3, b_5\}$
C <sub>18</sub>	$\{a_1, a_3, a_4, a_5\}$	$\{b_1, b_5\}$
C <sub>19</sub>	$\{a_2, a_3, a_4, a_5\}$	$\{b_3, b_5\}$
C <sub>20</sub>	$\{a_3, a_4, a_5, a_7\}$	$\{b_1, b_3\}$
C <sub>21</sub>	$\{a_1, a_3, a_4, a_5, a_7\}$	$\{b_1\}$
C <sub>22</sub>	$\{a_1, a_2, a_3, a_4, a_5\}$	$\{b_5\}$
C <sub>23</sub>	$\{a_2, a_3, a_4, a_5, a_7\}$	$\{b_3\}$
	$\{a_1, a_5, a_6, a_7\}$	$\{b_2\}$

**Algorithm 2** Lattice Creation

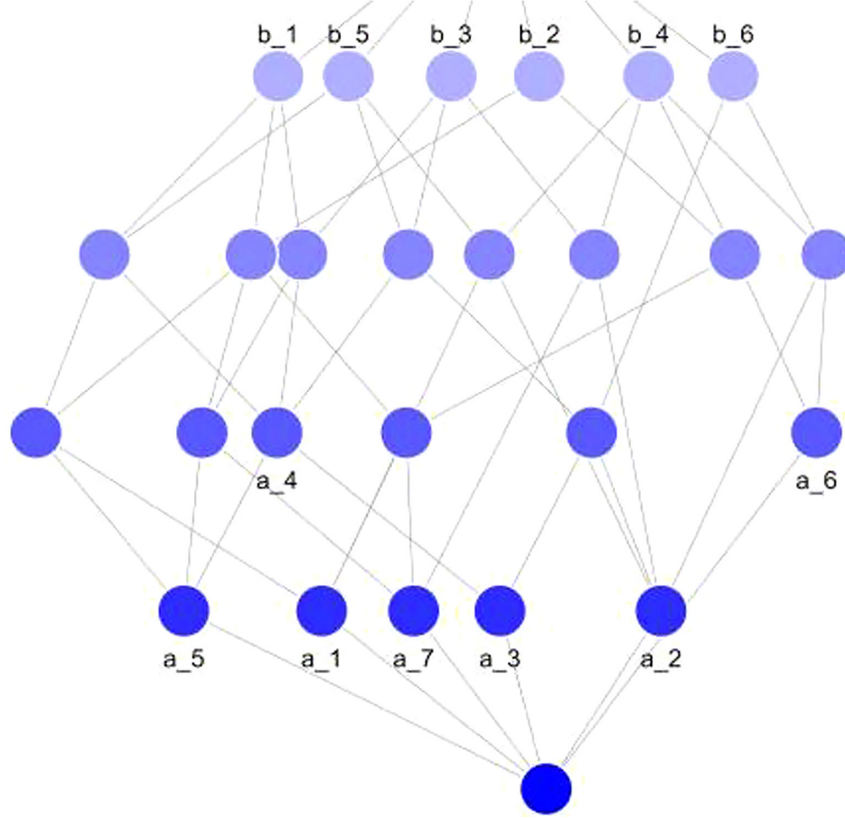
Graph as a lattice building from formal concepts.

Input: Set of formal concepts  $\Delta(\alpha, \beta)$

Output: Lattice  $\Gamma(\alpha, \beta)$

```

1: procedure CONSTRUCT— $\Gamma(\alpha, \beta)$ 
2:   for each formal concept  $\in \Delta(\alpha, \beta)$  do
3:     if  $((a_i > a_j) \ \&\& \ (b_i < b_j))$ 
4:       Place  $(a_i, b_i)$  as the super tuple of  $(a_j, b_j)$ 
5:       Create an edge from  $(a_j, b_j)$  which is hierarchically below to  $(a_i, b_i)$ .
6:   end for
7: end procedure
```



**Fig. 1.** Concept lattice formed from formal concepts.

It is highly important to note that the lattice generated is a poset (partially ordered set) with the following properties:

**Definition 6:** In the attribute set  $\beta$ , if  $\ll$  is the binary relationship within it, then  $(\beta, \ll)$  is regarded as a partially ordered set if;

$$b_j \ll b_i \quad (10)$$

(reflexive property)

$$b_i \ll b_j \& b_i \neq b_j \Rightarrow \neg b_j \ll b_i \quad (11)$$

(anti-symmetric property)

$$b_i \ll b_j \& b_j \ll b_k \Rightarrow b_i \ll b_k \quad (13)$$

(transitive property)

$$\forall b_i, b_j, b_k \in \beta \quad (14)$$

However, the exponential rate of growth of concepts in a concept lattice with the size of the context leads to a common difficulty in such systems. To solve the combinatorial issue, only important ideas or concepts are maintained by using a ranking system. It was in this context that the term “stability” was developed [22].

#### IV. A. STABILITY AND ITS ESTIMATION

The formal definition stability was first introduced in [22] and was updated later in [23]. It is defined as follows:

**Definition 7:** Suppose,  $(\alpha, \beta)$  be a formal concept of context  $(G, M, I)$ ; then, the stability (intensional) of the pair  $\eta(\alpha, \beta)$  is defined as:

$$\eta(\alpha, \beta) = \frac{|\{a_i \subseteq \alpha \mid a'_i = \beta\}|}{2^{|\alpha|}} \quad (15)$$

For example, if we want to analyze how stable something is, we need to know how many subsets there are of the concept extent, which is represented by  $X$ , whose description is identical to the concept intent (represented by  $B$ ). For example, how much a concept’s purpose relies on the concept’s objects (extent). Intensional stability is a synonym for this notion of stability. Stability is important because it allows us to see how things really are in the actual world, which is full of noise items. As an alternative definition, stability (extensional) might be described as follows: **Definition 8:** Similarly, suppose  $(\alpha, \beta)$  be a formal concept of the context  $(G, M, I)$ , then the stability (extensional) of the concept  $\eta_e(\alpha, \beta)$  is defined as:

$$\eta_e(\alpha, \beta) = \frac{|\{b_j \subseteq \beta \mid b'_j = \alpha\}|}{2^{|\beta|}} \quad (16)$$

It is important to note that the stability index is always between zero and one. Unfortunately, it is proven to be an NP-complete problem [26], and as a result, estimation of stability computation has previously been considered. Since  $L$  is the lattice’s size, this algorithm’s worst-case complexity  $(O(L^2))$  might expand exponentially with



the context's size. Space complexity becomes a problem when it comes to calculating a concept's stability since it requires a pre-calculation of its children's stability as well as the lattice's structural details. It might be beneficial to use proper estimate and approximation in order to calculate stability.

Buzmakov et al. [24] estimated stability in their study. They figured up an upper and lower limit for the stability index and combined them. A "bounding technique" is what they call it. However, the tightness of the binding cannot be guaranteed. [25]'s Monte-Carlo approximation for stability was also adopted by the authors. The bounding approach was shown to be more efficient than Monte-Carlo. It is called a "combined approach" since it incorporates both bounding and Monte-Carlo methods.

In this study, the index of stability is represented as a graph "GR," with the vertices and edges of the graph being labeled with particular language. By removing certain specific vertices and edges from the original graph, we can get a smaller graph  $\bar{GR}$ . The activity might result in more than one linked component. After that, we will add up the total number of cliques in  $\bar{GR}$  by counting how many are there in each linked connected component. We start off with some theorems.

Lemma 1:  $(\alpha, \beta)$  is defined as a formal concept of the context  $(G, M, I)$ ; then concept's stability index  $\eta(\alpha, \beta)$  can also be expressed as:

$$\eta(\alpha, \beta) = 1 - \frac{|\{a_i \subseteq \alpha \mid a'_i \neq \beta\}|}{2^{|\alpha|}} \quad (17)$$

Proof: Let  $Z$  be the set  $\{a_i \subseteq \alpha \mid a'_i = \beta\}$  and  $Z^\#$  be the set  $\{a_i \subseteq \alpha \mid a'_i \neq \beta\}$ .

It can be seen that  $Z$  and  $Z^\#$  are mutually exclusive (i.e. they cannot occur at the same time) and collectively exhaustive (i.e., their union must cover all the events within the entire sample space).

Thus,

$$\begin{aligned} Z &= \alpha - Z^\# \\ \Rightarrow \frac{|Z|}{2^{|\alpha|}} &= 1 - \frac{|Z^\#|}{2^{|\alpha|}} \\ \Rightarrow \frac{|\{a_i \subseteq \alpha \mid a'_i = \beta\}|}{2^{|\alpha|}} &= 1 - \frac{|\{X \subseteq \alpha \mid a'_i \neq \beta\}|}{2^{|\alpha|}} \\ \Rightarrow \eta(\alpha, \beta) &= 1 - \frac{|\{a_i \subseteq \alpha \mid a'_i \neq \beta\}|}{2^{|\alpha|}} \end{aligned}$$

from Definition 13

Using a graph model, we can now see how difficult it is to determine the stability of two concepts (A and B).

Let us assume  $GR = (N, D)$  is a graph bearing  $(N, D)$  as an ordered pair of the set of vertices and edges, respectively. Each object  $x_i$  of  $A$  for any concept  $(A, B)$  relates itself to a vertex  $N$  of the graph  $GR$ . Thus,  $N = \{x_1, x_2, \dots, x_n\}$  is the set of vertices or nodes. All the nodes of  $GR$  form a complete graph as mutually connected.

In order to map a formal concept  $(A, B)$  to a graph  $GR$ , the vertices of  $GR$  are clustered into two clusters:  $N_s$  and  $\bar{N}_s$ ; correspondingly the edges also form two clusters:  $D_s$  and  $\bar{D}_s$ . We now give the following definitions.

Definition 9: A vertex  $x_i \in N_s$ , if  $a'_i = B$ .

$$x'_i \neq B \Rightarrow x_i \in \bar{N}_s \quad (18)$$

Definition 10: An edge  $(x_i, x_j) \in D_s$ , if  $\{x_i, x_j\}' = B, i \neq j$ , otherwise  $(x_i, x_j) \in \bar{D}_s$ .

As an example, mapping concept  $C_{17}(\{a_3, a_4, a_5\} \{b_1, b_3, b_5\})$  as a graph 2 as a stability measure; where "unbroken-nodes" denote  $N_s$ , "broken-nodes" denote  $\bar{N}_s$ , "unbroken-lines" denotes  $D_s$ , and "broken-lines" represent  $\bar{D}_s$ .

A further classification over the set of  $\bar{N}_s$  vertices is done to form-  $\bar{N}_{s_e}$  and  $\bar{N}_{s_n} \cdot \bar{N}_{s_e}$  are incident to at least one  $\bar{D}_s$  edge. Also  $\bar{N}_{s_n}$  are the  $\bar{N}_s$  nodes not adjacent to any of  $\bar{D}_s$  edge.

Lemma 2: If  $(G, M, I)$  be a context,  $(\alpha, \beta)$  be a concept of the same. For any  $a_i \subseteq \alpha$ , if  $a'_i = \beta$ , then  $a'_j = \beta; \forall a_i \subseteq a_j \subseteq \alpha$ .

Proof: Given that

$$a_i \subseteq a_j \subseteq \alpha$$

$$\Rightarrow a' \subseteq a'_j \subseteq a'_i$$

(from Eq 4)

Since  $(\alpha, \beta)$  is given as a concept from  $(G, M, I)$  and  $a'_i = \beta$ ; then  $a'_j = \beta$  is valid.

Lemma 3: Edges generated between a  $N_s$  node and any of other nodes are always  $D_s$  edge type.

Proof: Let  $x_i$  be any  $N_s$  vertex such that  $x_i \subseteq g \subseteq \alpha$ . Now  $x'_i = \beta$  (by definition of  $N_s$ );  $\alpha' = \beta$  (by definition of concept). Thus  $g' = \beta$ .

As  $x_i \subseteq g$ ; there is an edge from each element (vertex) of  $g$  (except  $x_i$ ) to  $x_i$  and all the edges will be  $D_s$  edge (by definition of  $D_s$ ).

It is easily observable that  $D_s$  edges are created between the  $\bar{N}_{stb}$  vertices in consideration. We already know that 1-cliques are termed as vertices or nodes and 2-cliques are termed as edges between them. Formally, a clique of a given graph is the complete subgraph of the graph in question [27, 28]. The maximal clique is the size of the maximum complete graph present.

Lemma 4: Given  $(\alpha, \beta)$  as a concept of  $(G, M, I)$  and  $x_i$  is a subset of  $i$  items of  $\alpha$ . If there is at-least one  $D_s$  edge over the  $i$ -clique formed from  $x_i$ , then  $x'_i = \beta$ .

Proof: Given that  $(\alpha, \beta)$  is a concept and  $x_i$  is the subset of  $i$  objects of  $\alpha$ . If there is an  $E_s$  edge between  $a_i$  and  $a_j$  where  $a_i, a_j \in x_i$ , then  $\{a_i, a_j\}' = \beta$ . Also  $\alpha' = \beta$  and  $\alpha' \subseteq x' \subseteq \{a_i, a_j\}$ . Using Lemma 2;  $x'_i = \beta$ . Lemma 5: Let the graph  $GR$  be reduced to  $\bar{GR}$  by the removal of all  $N_s$  vertices and  $D_s$  edges. For any subset  $x_k$  of the reduced graph,  $x'_k \neq \beta$ .

Proof: As the graph formed consists of only  $\bar{N}_s$  vertices and  $\bar{D}_s$  edges; for any  $a_i$  and  $a_j \in x_k$  where  $i \neq j$ ;  $a_i' \neq \beta$ ,  $a_j' \neq \beta$  and  $\{a_i, a_j\}' \neq \beta$ . Using Lemma 2.;  $\alpha' \subseteq x_k \subseteq \{a_i, a_j\}$ . Thus,  $x_k' \neq \beta$ .

Lemma 6: The stability index computed from the reduced graph  $GR \subseteq GR$  after deletion of  $N_s$  vertices and  $D_s$  edges; will be compute the stability index identical to that calculated from the complete graph.

Proof: In order to compute the stability index of a graph  $GR$  (from Definition 13) we have to check the derivation of all the subsets in the power set of objects ( $2^{2^{|\alpha|}}$ ) for the concept (i.e the intent  $\beta$ ). Therefore, we have to select a set  $\chi$  of cliques from all possible cliques of size  $1, 2, \dots, |\alpha|$  in  $F$  that has at least one  $D_s$  edge (see Lemma 4.). Stability index is  $\frac{|\chi|}{2^{|\alpha|}}$ . Now, in the reduced graph  $GR$ , which consists of  $\bar{N}_s$  vertices and  $\bar{D}_{sb}$  edges, let  $\chi$  be the set of cliques of size  $1, 2, \dots, n$  and the derivative of all elements of  $\chi$  is not equal to intent

(see Lemma 5.). Stability index is  $1 - \frac{|\chi|}{2^{|\alpha|}}$ . Both the computed stabil-

ity index will be the same (see Lemma 1).

Theorem 1: For any  $(\alpha, \beta)$  defined as a formal concept over any formal context  $(G, M, I)$ ; the stability index  $\eta(\alpha, \beta)$  is at-least  $1 - \frac{\{|\alpha_c| + |\bar{N}_{se}| + 1\}}{2^{|A|}}$  where  $|\alpha_c|$  in a graph defines the maximum number of cliques.

Proof: In our case, in each connected component  $(\alpha_{ci})$  of the graph, as  $d \geq 1$ , there must be at-least one  $\bar{D}_s$  edge between two  $\bar{N}_s$  vertices. So, for each component,  $\bar{N}_{se}$  is the set of vertices ( $n$ ) and  $\bar{D}_s$  is connected to the vertices, the set of edges ( $m$ ) in a  $(n, m)$ - graph. The 0-clique is calculated redundantly for each connected component, so it must be deducted from each of it. Thus, the total number of maximal cliques formed will be:

$$\alpha_c = \sum_{n=1}^{|\alpha_c|} (K_{ci} - 1) \quad (19)$$

The remaining  $\bar{N}_{sn}$  vertices (having no edges, i.e.,  $d < 0$ ) have to be added with 1 (for 0-clique) along with it. Thus,

$$\left| \{X \subseteq \alpha \mid X' \neq \beta\} \right| \leq \left\{ |\alpha_c| + |\bar{V}_{stb_e}| + 1 \right\} \quad (20)$$

Then, finally, we get-

$$\eta(\alpha, \beta) \geq 1 - \frac{\{|\alpha_c| + |\bar{V}_{se}| + 1\}}{2^{|A|}} \quad (21)$$

Since counting the number of cliques in a graph is also an NP-hard problem, [29] estimated the maximum number of cliques in a graph, and it is pretty darn close. Degeneracy in a graph was utilized to assess how many cliques there were. There must be at least one vertex in every non-empty subgraph of a graph with  $d$  degrees to be called "d-degenerate." A graph's degeneracy is the minimal value of  $d$  that makes it d-degenerate. Degenerate graphs are (d-1)-degenerate, such as the full graph. K-degenerate is the term used to describe the graph's chromatic number. We use the following result from Wood [30]:

Result 1: In a  $(x, y)$ -graph; where  $|x|$  is the number of nodes and  $|y|$  is the number of edges.  $\forall 1 \leq d$ ; every d-degenerate graph GR with "x" vertices and  $\binom{d}{2} \leq y$  edges has at most-

$$\alpha_{ci} = x + \frac{(2^d - 1)y}{d} - \frac{(d-3)2^d + d + 1}{2} \quad (22)$$

cliques.

We finally forward an algorithm 3 for the estimated computation of the stability index.

## V. FORMAL ANALYSIS OF CONCEPTS IN ELECTRIC POWER SYSTEM

Electrical components are utilized to provide (produce), transmit, and consume electric power in an electric power system. An electric grid is a large-scale electric power infrastructure that provides electricity to households and businesses over a large area. Generation, transmission, and distribution are three layers of a three-layered complicated interdependent network. Control software and other equipment are also part of an electric grid's infrastructure, which transmits power from the source of generation to end customers. Transmission lines link generating buses to distribution substations, allowing for the transfer of electrical power [31]. The point of generating is generally placed in a central location that is far away from the point of consumption. In the power plant, electrical energy is produced by converting different forms of energy. Chemical, heat, hydraulic, mechanical, geothermal, nuclear, solar, and wind energy may all be derived from these types of energy sources. Electrical power can also be generated from any of these sources. It is then converted to high voltage, which is more appropriate for efficient long-distance transit to the consumption places through high voltage power lines, as a result of this conversion Transformers at the substations reduce the high voltage electrical energy to a lower voltage so

### Algorithm 3 Stability Computation

Compute  $\eta$ ,  $\forall (\alpha, \beta)$  of a given context  $(G, M, I)$

Input: Formal Context  $(G, M, I)$

Output:  $\eta(\alpha_i, \beta_j) \forall$  Concept set  $\Delta$

```

1: procedure COMPUTE-STABILITY INDEX
2:   for each  $\Delta(\alpha, \beta) \in (G, M, I)$  do
3:     Compute set of  $\bar{N}_s$  ( $\bar{N}_{se}$  and  $\bar{N}_{sn}$ ) and  $\bar{E}_{stb}$  at Algorithm1.
4:     Compute all components which are totally connected (TC) of the graph using  $\bar{N}_{se}$ 
       and  $\bar{D}_s$  as the set of vertices and edges respectively [?].
5:     for each TC do
6:       Compute the adjacency matrix  $AM$  of the nodes.
7:       Compute  $\lambda$ .
8:       Compute  $\alpha_{ci}$  (Result 1)
9:     end for
10:    Compute  $1 - \frac{\{|\alpha_c| + |\bar{N}_{se}| + 1\}}{2^{|A|}}$ 
11:    Compute the maximum stability index. (Theorem 1.)
12:  end for
13: end procedure
    
```

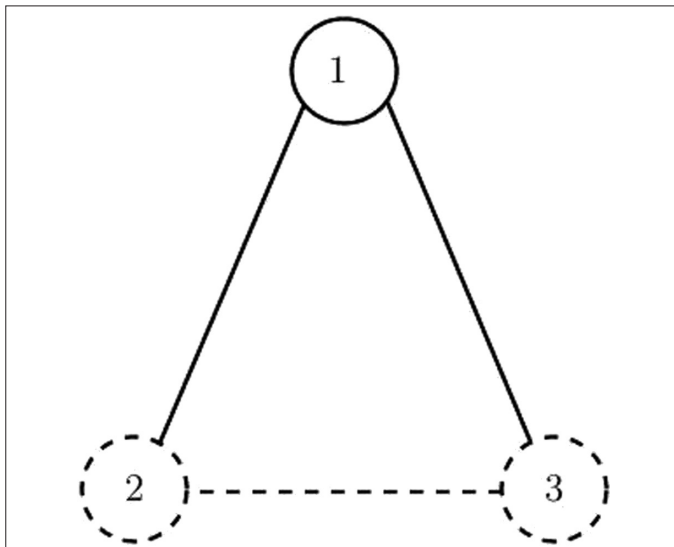
that it may be distributed for residential, commercial, and industrial use. In order to minimize the network's capacity, a transformer is used to scale down voltages to lower levels for the end user. One source of power is sufficient for the grid, but it is generally interconnected

**TABLE IV.** FORMAL CONTEXT WITH REGARD TO COMPONENT AND FAILED STATE

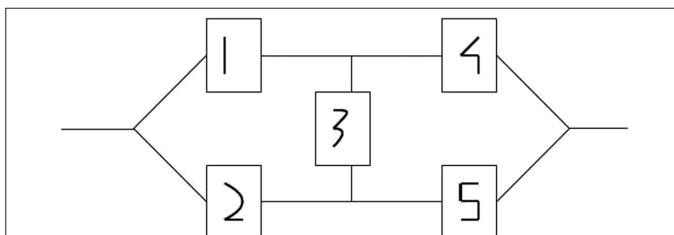
	$s_1$	$s_2$	$s_3$	$s_4$	$s_5$	$s_6$	$s_7$	$s_8$	$s_9$
$c_1$	X	X		X	X		X	X	
$c_2$			X	X	X	X			X
$c_3$	X	X			X	X		X	
$c_4$	X					X			
$c_5$	X	X	X		X		X		X

to additional sources to give a broader range of options and greater reliability.

Real-time energy delivery implies that electricity is created when the power switch is switched on, transferred, and provided. For these systems, there is no need to store electrical energy but rather to create electricity when the need arises. All electrical power systems are designed to function under reasonably consistent weather and loading conditions. Due to harsh weather, these design assumptions might be challenged. High-voltage transformers, both within and outside the substations, are the most sensitive electrical equipment to catastrophic damage. Transmission lines rely on these enormous, heavy, and difficult-to-move transformers at substations. In order to



**Fig. 2.** Mapping concept  $C_{17}(\{a_3, a_4, a_5\} \{b_1, b_3, b_5\})$  to a graph.



**Fig. 3.** A component network diagram.

replace most of these specially manufactured gadgets, the delivery period might be rather extensive. There are also other natural catastrophe risks, including damage to generators and transmission lines. In the event of a natural catastrophe, fuel supplies might be disrupted and transmission lines may be damaged. Disasters that impair a control center that coordinates the grid's functioning may have a significant influence on the grid's dependability. When it comes to transient stability issues in power systems, cut-sets play an important role.

In graph theory and network research, the idea of cut-sets is crucial [32]. When a graph is divided into two separate subgraphs, a cut-set is a collection of edges that may be deleted to accomplish this. A large number of notions and features in graphs and networks are strongly associated with cut-sets. As an example, a graph's edge connectivity is defined as the minimal cut-set's cardinality. The max-flow min-cut theorem [1] states that the entire capacity of a flow network's minimal cut-set is equal to the maximum amount of flow going from the source to the sink. As a combinatorial optimization problem with theoretical importance and a broad variety of applications, finding the largest cut-set has been intensively researched in discrete mathematics.

There are two types of cut-sets in reliability analysis: those that can fail and those that cannot. It is important to find the minimum cut-set,

**TABLE V.** GENERATED CONCEPTS FROM COMPONENT AND STATE CONTEXT

Concept <sub>i,d</sub>	A	B
Co <sub>1</sub>	$\{c_1, c_2, c_3, c_4, c_5\}$	$\phi$
Co <sub>2</sub>	$\{c_2, c_5\}$	$\{s_3, s_5, s_9\}$
Co <sub>3</sub>	$\{c_1, c_3\}$	$\{s_1, s_2, s_5, s_8\}$
Co <sub>4</sub>	$\{c_1, c_5\}$	$\{s_1, s_2, s_5, s_7\}$
Co <sub>5</sub>	$\{c_1, c_2\}$	$\{s_4, s_5\}$
Co <sub>6</sub>	$\{c_2, c_3, c_4\}$	$\{s_6\}$
Co <sub>7</sub>	$\{c_1, c_3, c_5\}$	$\{s_1, s_2, s_5\}$
Co <sub>8</sub>	$\{c_1, c_2, c_3, c_5\}$	$\{s_5\}$
Co <sub>9</sub>	$\{c_1, c_3, c_4, c_5\}$	$\{s_1\}$
Co <sub>10</sub>	$\{c_2, c_3\}$	$\{s_5, s_6\}$
Co <sub>11</sub>	$\{c_3, c_4\}$	$\{s_1, s_6\}$
Co <sub>12</sub>	$\{c_1\}$	$\{s_1, s_2, s_4, s_5, s_7, s_8\}$
Co <sub>13</sub>	$\{c_3\}$	$\{s_1, s_2, s_5, s_6, s_8\}$
Co <sub>14</sub>	$\{c_5\}$	$\{s_1, s_2, s_3, s_5, s_7, s_9\}$
Co <sub>15</sub>	$\{c_2\}$	$\{s_3, s_4, s_5, s_6, s_9\}$
Co <sub>16</sub>	$\phi$	$s_1, s_2, s_4, s_5, s_6, s_7, s_8, s_9\}$



which is any cut set that does not include any other as a subset and must fail to cause system failure in order to be considered reliable [33].

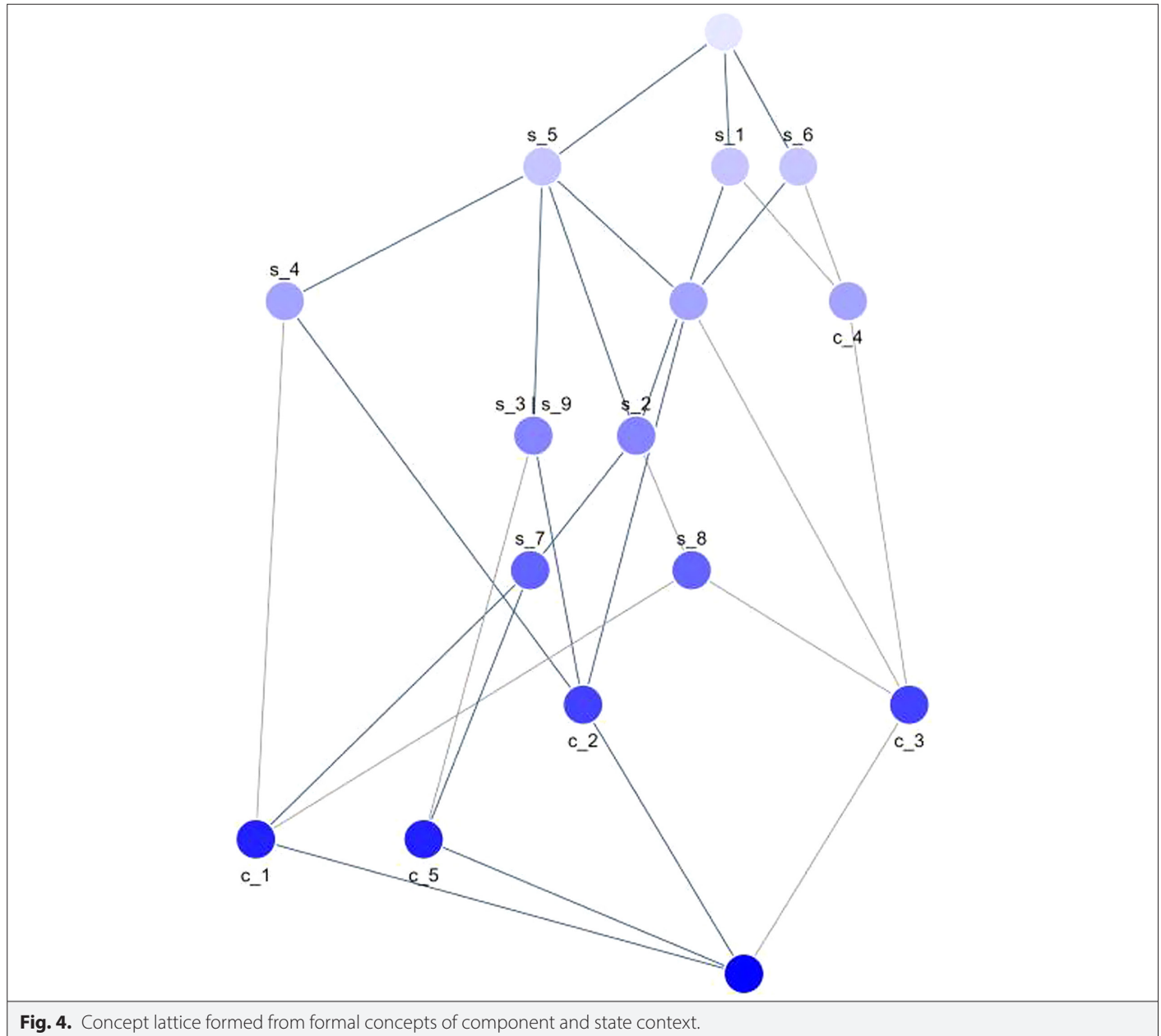
These five components are shown in a dependability block diagram in Fig. 3. Assuming that the  $(x,y)$  is a reference to the two failed components, we know that there are four minimal cut-sets:  $CS_1=(c_1,c_2)$ ;  $CS_2=(c_4,c_5)$ ,  $CS_3=(c_1,c_3,c_5)$ , and  $CS_4=(c_2,c_3,c_4)$ . Each set's ranking is determined by its cardinality. Third-order minimal cut-set  $CS_3=(c_1,c_3,c_5)$  is an example. Low-order minimum cut sets are not included in high-order minimum cut-sets. As an example  $c_1$  and  $c_2$  are not included in  $CS_3$ . A total of  $2^5$  fail-states could be generated as a result.

Consider Table IV to provide an illustration of the FCA technique. This table depicts the 9 failed states  $s_i$  of the system  $\{s_1,s_2,\dots,s_9\}$  (the remaining 23 states (32—9) relate to operating states and are not shown here). The formal setting of our case will be defined by

the table. It is important to note that, as previously said, the table defines the connection between components and the states of the system: if a component fails, it belongs to a state; otherwise, it does not. Taking the example of state  $s_5$ , the components  $c_1,c_2,c_3$  and  $c_5$  all fail. Figure 2 shows an example of a graph created for the stability computation of a concept.

Table V denotes the 16 concepts that were created by algorithm 1 and the generated lattice for the concepts is shown in Fig. 4.

When examined, there are four minimal cut-sets:  $S_1=(c_1,c_2)$ ;  $S_2=(c_4,c_5)$ ,  $S_3=(c_1,c_3,c_5)$ , and  $S_4=(c_2,c_3,c_4)$  has the failed state common as  $(s_4,s_5)$ ,  $(s_1)$ ,  $(s_1,s_2,s_5)$  and  $(s_6)$ . This can be seen when we move up the lattice with respect to the components. Also, if we move down the lattice, we can find because of which component, the fail-state arrived.



**Fig. 4.** Concept lattice formed from formal concepts of component and state context.

## VI. CONCLUSION

It is the purpose of this work to introduce the reliability practitioner to FCA and to demonstrate how the technique may be simply implemented in particular reliability-related scenarios. However, both fields of study gain from the applications. Structure, relationships, and visualization are all keys to FCA's data analysis approach. It is all about the context table for FCA: (objects, attributes, and binary relation details). A clear graphical representation of the data structure is produced by FCA (formal concepts and order relations). Theorem, lemmas, and definitions were presented with proper proofs. A case study in the field of electric power system is covered in the article. A binary connection between a set of objects and a set of characteristics is analyzed by FCA in instances where this is practicable. For example, in the case of fault diagnostic evaluation, the components involved in a particular occurrence are specified. The FCA technique may be used for a wide range of applications, including threat assessment, community identification in networks, and software reliability, among others.

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