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Adaptive Predictive Control of Fractional Order Chaotic Systems

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ABSTRACT

In this paper, an adaptive predictive control for controlling and stabilizing fractional order chaotic systems around the equilibrium point is provided. The stability of fractional order chaotic systems around equilibrium points in the presence of parameter uncertainty has been demonstrated using Lyapunov's stability theorem. In addition, the uncertain parameters of fractional order chaotic systems are calculated using appropriate adaptive methods based on the proposed predictive controller structure. Rossler and Chen systems were considered to numerical simulations. The results demonstrated the adaptive predictive control method's usefulness and performance.

Index Terms—Adaptive control, equilibrium point, fractional order chaotic systems, predictive control.

I. INTRODUCTION

Chaotic systems are nonlinear systems that are extremely sensitive to initial conditions and are difficult to predict. These systems have been observed in a variety of applications including chemistry, biology, finance, and engineering. Many control strategies, such as adaptive control [1–5], sliding mode control [6, 7], adaptive back-stepping control [8], and predictive and fuzzy control [9], have been applied in research to govern chaotic systems. In recent decades, scholars have been particularly interested in fractional calculations. These calculations are related to non-integer derivatives (fractional). Because fractional calculations have a memory and inheritance property, various systems are explained more correctly than integer order models. Controlling chaotic systems with a fractional order model is one of the newest topics of interest among researchers. Many controllers were utilized in the research to control fractional chaotic systems.

The followings are among the studies on the use of predictive control to govern the behavior of fractional chaotic systems. In [10], the chaotic system is controlled by a fuzzy and predictive controller. The authors of [11] examined the dynamics of a fractional order chaotic system, its stabilization via predictive control, and its circuit validation. In 2019, Zoad et al. [12] used predictive control of a fractional order delayed chaotic system with its circuit implementation.

Wang et al. [13], in 2018, proposed a nonlinear fuzzy predictive control for a class of integer order chaotic systems. In 2017, Khan et al. [14] employed a predictive controller to control the Rabinovich chaotic system. The control input was arranged in such a way that the chaotic trajectories converged on the unstable equilibrium points. A practical predictive control model for a set of noisy chaotic hybrid systems connected to the Chua circuit was proposed in [15]. Wang et al. suggested a fuzzy generalized predictive control for fractional order nonlinear systems in 2017 [16]. Based on the Grünwald–Letnikov definition, Laplace transform, and discretization, a group of fractional order nonlinear systems was translated to the autoregressive community moving average (CARMA) model in the stated article. A linear CARMA model for nonlinear systems was presented based on Takagi–Sugeno's fuzzy theory. Then, using the CARMA predictive model and generalized predictive control theory, a generalized predictive control approach for fractional order nonlinear systems was proposed. To control chaotic systems, [17] employed a robust fractional order controller based on predictive control. The simulation results on three-dimensional Lorenz and Chen chaotic systems demonstrated the efficacy of the robust control technique. In [18], Zheng and Li used predictive control to govern fractional order systems.

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In a class of fractional chaotic systems, Mofid et al. exploited observer control of sliding mode disturbance based on adaptive synchronization [19]. In [20], an active control technique was used to build a robust fractional adaptive intelligent controller for uncertain fractional order chaotic systems. Mirzajani et al. created a T-S adaptive fuzzy controller for fractional order systems with uncertain parameters and input limitations. The properties of the unknown fractional system were taken into account in their control strategy. The adaptive procedures were then built to estimate these parameters using Lyapunov's stability theorem [21].

To prevent chaos in fractional systems, Paneh Kolai et al. used an adaptive fractional mode feedback controller. For reduced and full mode feedback controllers with singular and vector feedback gains, they adopted a different control approach [22]. Lu et al. [23] suggested finite-time adaptive neural network control for the fractional order chaotic system of a permanent magnet synchronous motor in 2020.

The economic chaotic fractional order system was controlled using an active control in [24]. Shukla and Sharma used Lyapunov and Mittag–Leffler stability theorems to stabilize a group of fractional chaotic systems utilizing the back-stepping approach [25]. Ni et al. in 2017 applied the non-singular time-constant fractional-order terminal sliding mode synchronization approach for the control of fractional-order chaotic systems [26]. To control fractional order integrated chaotic systems in presence of external disturbances, [27] employed fractional integral sliding surface control with a convolutional algorithm. The authors of [28] proposed a new fractional order memristor 3D chaotic system. The topic of control and synchronization was then investigated using PI control in sliding mode.

Deep recurrent neural networks with finite-time terminal sliding mode control for a chaotic fractional order financial system with market certainty were investigated in [29]. The authors of [30] investigated chaotic dynamics and chaotic control for a time-delayed fractional order satellite model. For chaos control, a simple linear feedback control mechanism was adopted in this paper. The authors of [31] used feedback and adaptive control techniques to control and synchronize fractional order chaotic systems. The authors of [32] described a novel method for chaotic modeling, control, and synchronization of a fractional biological oscillator. For memory modeling in the system, the concept of fractional order was applied. The authors of [33] investigated the dynamics and synchronization control of symmetric fractional order chaotic systems. The authors of [34] described the stability of a fractional order system with uncertainty and disturbance. A basic linear feedback control strategy was utilized to control fractional chaotic systems in this paper. Zouad et al. in 2019, designed a new secure communication scheme for fractional order delayed chaotic system, then they simulated of its electronics circuit [35].

Because the parameters of chaotic systems are often unknown in practice, an adaptive control technique is required to estimate the system parameters. The predictive controller was also adopted because of its ability to predict the system's future behavior. As a result, the fundamental contribution of this paper is the combination of predictive control and adaptive control to control the behavior of chaotic systems with a fractional order model toward the equilibrium point. A previous study indicates that the combination of these two controllers has not been examined in this application. In this article, the parameter estimation rules are constructed using Lyapunov's stability theorem, and the system states converge to the equilibrium point using predictive control.

The following is the rest of this article. The basic concepts and preliminary steps for fractional calculations are covered in the second section. The third section describes how to develop predictive and adaptive control for fractional order chaotic systems. The fourth section includes numerical simulations. The fifth section contains conclusions and recommendations for future research in this topic.

II. PRELIMINARIES

For fractional calculus, there are three well-known definitions: Caputo, Riemann–Liouville, and Grünwald–Letnikov. For fractional calculations, the Grünwald–Letnikov definition is used in this article. The fractional derivative using Grünwald–Letnikov concept is as follows [28]:

$$ {}_aD_t^{\alpha} f(t) = \lim_{h\to 0} \frac{1}{h^{\alpha}} \sum_{j=0}^{[(t-a)/h]} (-1)^j \binom{\alpha}{j} f(t-jh), \tag{1}$$

where ${}_aD_t^{\alpha} f(t)$ is the derivative-integral operator of fractional order with the order α ($\alpha \in R$), which means the integer part, $(t-a)/h$, and a and t are operational ranges for the operator ${}_aD_t^{\alpha} f(t)$. The expression $\binom{\alpha}{j}$ is given as follows:

$$\binom{\alpha}{j} = \frac{\alpha!}{j!(\alpha-j)!} = \frac{\Gamma(\alpha+1)}{\Gamma(j+1)\Gamma(\alpha-j+1)} \tag{2}$$

where Γ is the gamma function.

For numerical simulation, the modified version of Eq. (2) is used. The numerical approximation of α at kh points ($k=1, 2, \ldots$) is as follows.

$$ (k-L_m/h) D_{t_k}^{\alpha} f(t) \approx \frac{1}{h^{\alpha}} \sum_{j=0}^{k} (-1)^j \binom{\alpha}{j} f(t_k - j), \tag{3}$$

where L_m is the memory length, $t_k = kh$, h is the time step, $(-1)^j \binom{\alpha}{j}$ are binomial coefficients, and $c_j^{\alpha} (j = 0,1,2,...)$

$$ c_0^{(\alpha)} = 1, c_j^{(\alpha)} = (1 - \frac{1+\alpha}{j}) c_{j-1}^{(\alpha)}, \tag{4}$$

III. PROBLEM STATEMENT AND CONTROLLER DESIGN

In this part, first, a predictive feedback control is devised to stabilize a fractional order chaotic system at the equilibrium point by introducing a control signal to the fractional order chaotic system. The adaptive controller is then employed to estimate the uncertain parameters of the fractional order chaotic system. In this paper, it has been assumed states of the system are observable.

A. Predictive Feedback Controller Design

The controlled system is defined as follows:

$$ D^{\alpha} x(t) = f(x(t)) + u(t) \tag{5}$$

Assume that the control law is as follows:

$$u(t)=k\left(D^{\alpha}x(t)+x(t)-x_f\right)=k\left(f\left(x(t)\right)+x(t)-x_f\right), \quad (6)$$

in which x_f is an unstable equilibrium point of the system (5) and k is a negative control parameter such that $k \neq -1$. By substituting Eq. (6) in Eq. (5), we have:

$$D^{\alpha}x(t)=\tilde{f}(x(t))=(1+k)f(x(t))+k(x(t)-x_f) \quad (7)$$

Assume that $\lambda_1, \lambda_2, \ldots, \lambda_m$ are the eigenvalues of the Jacobian matrix (D_f) of the system (5) in the equilibrium point x_f without controller. Also assume that σ_j,β_j are the real and imaginary parts of the eigenvalues of the Jacobian matrix of system (5) without controller. The authors of [18] have shown that the following equation holds:

$$\bar{k}=\min_{1\le j\le n}\{\frac{-\sigma_j\tan\frac{\alpha\pi}{2}+|\beta_j|}{(1+\sigma_j)\tan\frac{\alpha\pi}{2}-|\beta_j|}\}| \quad |\arg(\sigma_j+\beta_j i)|\le\frac{\alpha\pi}{2}\}. \quad (8)$$

Assuming that $\bar{k}$ is equal to Eq. (8), it was shown in [18] that if $k\in(-1,\bar{k})$, then the equilibrium point, $x_f \in R^n$, of the controlled system (5) is asymptotically locally stable. To illustrate this, consider the following theorem:

Theorem 1 [*18*]: *If the control law u(t) is considered as Eq. (6), then the system of Eq. (7) will be stable towards the equilibrium point, x_f, considering $k \neq -1$.*

Note 1: Since the stability around the equilibrium point is guaranteed, the predictive control law is defined as the following switching law:

$$u(t)=\begin{cases}k(f(x)+x-x_f) & if \quad \|x((t-1)-x(t)\|<\varepsilon \\ 0 & otherwise,\end{cases} \quad (9)$$

where Ɛ is a small positive number such that it meets conditions of Eq. (9).

A Lyapunov function and its derivative are considered as follows:

$$V=\frac{1}{2}e^2, D^{\alpha}V=eD^{\alpha}e \quad (10)$$

In which, the control error toward the equilibrium point, $e=x-x_f$, is zero. By applying of the controller (6) to system (5) and applying the fractional order derivative to the Lyapunov function in Eq. (10), the following equation is obtained.

$$D^{\alpha}V=e(f(x)(1+k)+ke), k<0, k\neq-1, (1+k)\sigma_j+k>0, 1+k+\eta\approx 0 \quad (11)$$

where η is a small positive number. The Laypunov's derivative is as follows:

$$D^{\alpha}V=e(f(x)(1+k)+ke)\approx e(ke+\eta f(x))<0, \quad (12)$$

where $\eta_f(x)$ is a small number close to zero. Therefore, in terms of the Lyapunov stability criterion, the stability conditions are also established because of η being small. Now, if there are parameters with uncertainty in the system (5), this control method must have fundamental changes, therefore, the innovation of this article is the use of the adaptive control plan in conditions where the system parameters are unknown.

Remark. Control law in Eq. (6) can be applied in practical applications. The controller is generally a negative feedback of the observed system states $x(t)$ and the known equilibrium point x_f. k is a negative expression and it is calculated by implementation of predictive control. The expression $f(x(t))$ may have unknown parameters that will be obtained by using adaptive rules.

B. Designing of the Adaptive Controller Combined With Predictive Control

According to the predictive controller designed in the previous section, an adaptive controller is proposed in this section to estimate the unknown parameters of fractional order chaotic systems.

It is considered that a hyper-chaotic system with a controller can be generalized using Eq. (13).

$$\begin{cases}D^{\alpha}x_1=A_1x+g_1(x)+u_1(t)\\ D^{\alpha}x_2=A_2x+g_2(x)+u_2(t)\\ D^{\alpha}x_3=A_3x+g_3(x)+u_3(t)\\ D^{\alpha}x_4=A_4x+g_4(x)+u_4(t)\end{cases} \quad (13)$$

In which A_1,A_2,A_3,A_4 include uncertain parameters. The terms $g_i(x)$, $i=1, 2, 3, 4$, and $u_i(t)$ include the state variables without parametric uncertainty and control efforts, respectively. The control efforts are calculated as in Eq. (14).

$$\begin{cases}u_1(t)=k_1(\breve{A}_1x+g_1(x(t)+(x_1-x_{f1}))\\ u_2(t)=k_2(\breve{A}_2x+g_2(x(t)+(x_2-x_{f2}))\\ u_3(t)=k_3(\breve{A}_3x+g_3(x(t)+(x_3-x_3))\\ u_4(t)=k_4(\breve{A}_4x+g_4(x(t)+(x_4-x_{f4}))\end{cases} \quad (14)$$

In Eq. (14), $k_i, i=1, 2, 3, 4$ are negative. $\breve{A}_1,\breve{A}_2,\breve{A}_3,\breve{A}_4$ are parameter estimations of system (13) determined in Eq. (18). The estimation error is defined as $\tilde{A}_i=\breve{A}_i-A,\ i=1,2,3,4$. The x_{fi} parameters are the components of the equilibrium point and equal to zero.

Theorem 2*: If the control signal (14) is applied to the system (13) along with the parameter estimation criteria in Eq. (18), the system (13) will tend to the equilibrium point asymptotically, according to the Lyapunov stability theorem.*

Proof: The Lyapunov function is used to control the system toward the equilibrium point as Eq. (15).

$$V=\frac{1}{2}e_1^2+\frac{1}{2}e_2^2+\frac{1}{2}e_3^2+\frac{1}{2}e_4^2+\frac{1}{2}\tilde{A}_1^2+\frac{1}{2}\tilde{A}_2^2+\frac{1}{2}\tilde{A}_3^2+\frac{1}{2}\tilde{A}_4^2 \quad (15)$$

The proposed Lyapunov function's derivative is as in Eq. (16):

$$
\begin{aligned}
D^{\alpha}V &= e_1(D^{\alpha}e_1)+e_2(D^{\alpha}e_2)+e_3(D^{\alpha}e_3)+e_4(D^{\alpha}e_4)\\
&\quad+\tilde{A}_1(D^{\alpha}\tilde{A}_1^2)+\tilde{A}_2(D^{\alpha}\tilde{A}_2^2)+\tilde{A}_3(D^{\alpha}\tilde{A}_3^2)+\tilde{A}_4(D^{\alpha}\tilde{A}_4^2)\\
&= e_1(A_1x+g_1(x)+u_1)+e_2(A_2x+g_2(x)+u_2)+\\
&\quad e_3(A_3x+g_3(x)+u_3)+e_4(A_4x+g_4(x)+u_4)+\\
&\quad \tilde{A}_1(D^{\alpha}\tilde{A}_1)+\tilde{A}_2(D^{\alpha}\tilde{A}_2)+\tilde{A}_3(D^{\alpha}\tilde{A}_3)+\tilde{A}_4(D^{\alpha}\tilde{A}_4)
\end{aligned} \tag{16}
$$

The derivative of the Lyapunov function will be as follows after applying of the controller (14) in (16):

$$
\begin{aligned}
D^{\alpha}V &= e_1(x(1+k_1)A_1+k_1\tilde{A}_1x+k_1e_1)+\\
&\quad e_2(x(1+k_2)A_2+k_2\tilde{A}_2x+k_2e_2)+\\
&\quad e_3(x(1+k_3)A_3+k_3\tilde{A}_3x+k_3e_3)+\\
&\quad e_4(x(1+k_4)A_4+k_4\tilde{A}_4x+k_4e_4)+\\
&\quad \tilde{A}_1(D^{\alpha}\tilde{A}_1)+\tilde{A}_2(D^{\alpha}\tilde{A}_2)+\tilde{A}_3(D^{\alpha}\tilde{A}_3)\\
&\quad +\tilde{A}_4(D^{\alpha}\tilde{A}_4)
\end{aligned} \tag{17}
$$

If parameter estimation rules are taken into account like (18):

$$
\begin{aligned}
&D^{\alpha}\tilde{A}_1=-k_1e_1x+\lambda_1e_{A_1},D^{\alpha}\tilde{A}_2=-k_2e_2x+\lambda_2e_{A_2},\\
&D^{\alpha}\tilde{A}_3=-k_3e_3x+\lambda_3e_{A_3},D^{\alpha}\tilde{A}_4=-k_4e_4x+\lambda_4e_{A_4},
\end{aligned} \tag{18}
$$

where $\lambda_i < 0, i = 1, 2, 3, 4$. By applying the parameter estimation laws and considering that $x(1+k_i)A_i$, $I = 1, 2, 3, 4$ is close to zero, the derivative of the Lyapunov function becomes as follows:

$$
D^{\alpha}V = k_ie_i^2+\lambda_i\tilde{A}_i^2, i=1,2,3,4 \tag{19}
$$

As a result, the fractional derivative of Lyapunov function is negative, and the parameters are determined using Eq. (18). Fig. 1 depicts the block diagram of the adaptive predictive control method for chaotic system control.

As shown in Fig. 1, the uncertain parameters are estimated in the adaptive design block before entering the predictive control block and calculating the control effort vectors. The control effort is then applied to the chaotic system in order to stabilize and converge the system states toward the equilibrium point. The states of X are calculated and compared to the reference value to determine the error. The pre-control procedure between the systems then continues using the obtained error and the estimated parameters. This process is repeated until the simulation time is over and the states converge to the equilibrium point.

In the following, the simulations and the results on two fractional order chaotic systems of Rossler and the Chen hyper-chaotic system.

IV. SIMULATIONS AND RESULTS

The proposed adaptive predictive controller was applied to Rossler chaotic system and Chen hyperchaotic systems in this section, and the results were obtained. The simulation findings for Rossler system are presented first, followed by the simulation results for Chen's hyperchaotic system.

A. Rossler System

The fractional order Rossler system is described as follows:

$$
\begin{cases}
D^{\alpha}x_1(t)=-x_2(t)-x_3(t),\\
D^{\alpha}x_2(t)=x_1(t)+ax_2(t),\\
D^{\alpha}x_3(t)=bx_1(t)-(c-x_1(t))x_3(t),
\end{cases} \tag{20}
$$

This system with order $\alpha = 0.97$ and $a = 0.34$, $b = 0.4$, $c = 4.5$ shows chaotic behavior [18].

Consider the system with controller $u(t)=[u_1 \quad u_2 \quad u_3]^T$, as follows:

$$
\begin{aligned}
&D^{\alpha}x_1(t)=-x_2(t)-x_3(t)+u_1(t),\\
&D^{\alpha}x_2(t)=x_1(t)+ax_2(t)+u_2(t),\\
&D^{\alpha}x_3(t)=bx_1(t)-(c-x_1(t))x_3(t)+u_3(t),
\end{aligned} \tag{21}
$$

Assume that the parameters a, b, and c are uncertain and must be estimated. Control errors relative to the equilibrium point are defined as $e_i = x_i - x_{f_i}, i = 1,2,3$. It is shown in [18] that x_{f_i} is at the origin.

In this system it is defined that $\tilde{a}=\breve{a}-a$, $\tilde{b}=\breve{b}-b$, $\tilde{c}=\breve{c}-c$, . Therefore, $D^{\alpha}\tilde{a}=D^{\alpha}\breve{a}, D^{\alpha}\tilde{b}=D^{\alpha}\breve{b}, D^{\alpha}\tilde{c}=D^{\alpha}\breve{c}$, . According to Theorem 2, the controllers can be calculated as follows:

$$
\begin{aligned}
&u_1=k_1(-x_2-x_3+e_1)\\
&u_2=k_2(x_1+\breve{a}x_2+e_2)\\
&u_3=k_3(\breve{b}x_1-\breve{c}x_3+x_1x_3+e_3)
\end{aligned} \tag{22}
$$

where k_1, k_2, $k_3 = -0.95$. Estimation rules for unknown parameters are written as follows:

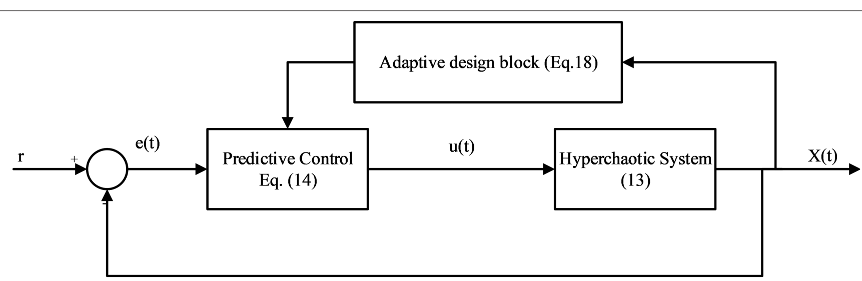

Fig. 1. Block diagram of the proposed scheme for adaptive predictive control for controlling chaotic systems

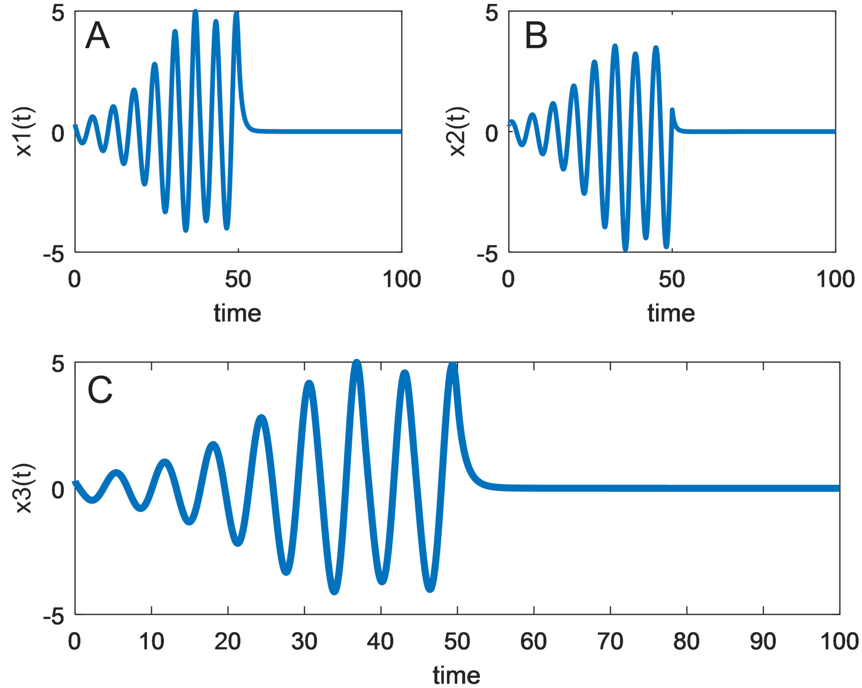

Fig. 2. (a) $x_1(t)$, $x_2(t)$, $x_3(t)$ vs. time

$$D^{\alpha}\breve{a} = -k_2 e_2(x_2) + \lambda_a e_a$$
$$D^{\alpha}\breve{b} = -k_3 e_3 x_1 + \lambda_b e_b \qquad (23)$$
$$D^{\alpha}\breve{c} = k_3 e_3 x_3 + \lambda_c e_c$$

where λ_a, λ_b, and $\lambda_c = -0.95$.

B. Fractional Order Hyperchaotic Chen System

Consider the fractional order hyperchaotic Chen system, which is as follows:

$$\begin{cases} D^{\alpha}x_1(t) = a(x_2(t) - x_1(t)) + x_4(t), \\ D^{\alpha}x_2(t) = dx_1(t) - x_1(t)x_3(t) + cx_2(t), \\ D^{\alpha}x_3(t) = x_1(t)x_2(t) - bx_3(t), \\ D^{\alpha}x_4(t) = x_2(t)x_3(t) + rx_4(t), \end{cases} \qquad (24)$$

The system with control input is defined as follows:

$$D^{\alpha}x_1(t) = a(x_2(t) - x_1(t)) + x_4(t) + u_1(t),$$
$$D^{\alpha}x_2(t) = dx_1(t) - x_1(t)x_3(t) + cx_2(t) + u_2(t), \qquad (25)$$
$$D^{\alpha}x_3(t) = x_1(t)x_2(t) - bx_3(t) + u_3(t),$$
$$D^{\alpha}x_4(t) = x_2(t)x_3(t) + rx_4(t) + u_4(t),$$

Chaos control errors are $e_i = x_i$, $i = 1, 2, 3, 4$. Uncertain parameters include a, b, c, d, and r and must be estimated. In this system, parameter estimation errors are as $\tilde{a} = \breve{a} - a, \tilde{b} = \breve{b} - b, \tilde{c} = \breve{c} - c, \tilde{d} = \breve{d} - d, \tilde{r} = \breve{r} - r$. According to Theorem 2, the control efforts are calculated as follows:

$$u_1 = k_1(e_1 + \breve{a}(x_2 - x_1) + x_4)$$
$$u_2 = k_2(e_2 + \breve{d}x_1 - x_1x_3 + \breve{c}x_2) \qquad (26)$$
$$u_3 = k_3(e_3 + x_1x_2 - \breve{b}x_3)$$
$$u_4 = k_4(e_4 + x_2x_3 + \breve{r}x_4)$$

In which $k_1 = k_2 = k_3 = k_4 = -0.95$ are the predictive control gains. According to Eq. (18), the adaptive laws of the parameters are determined as follows:

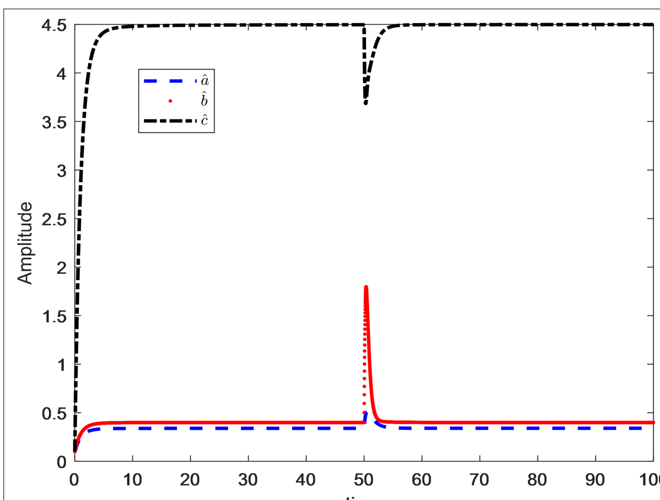

Fig. 3. $\breve{a}, \breve{b}, \breve{c}$ estimations vs. time

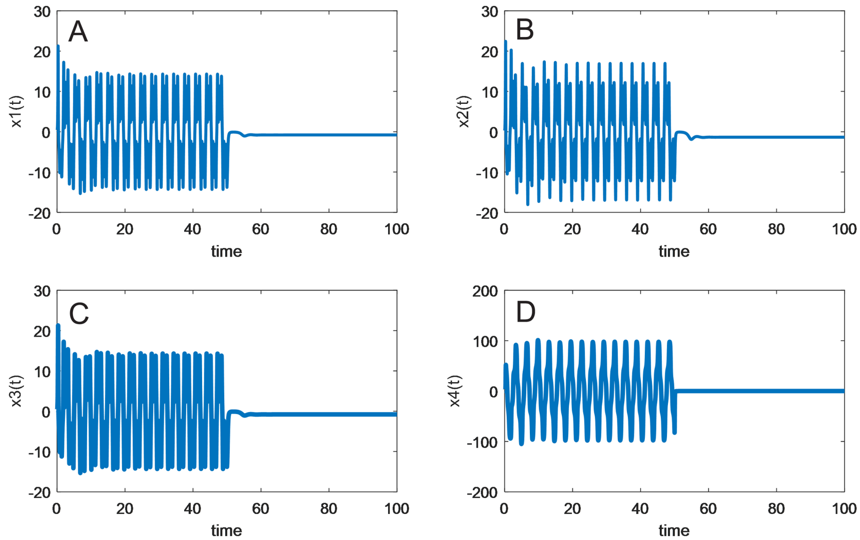

Fig. 4. $x_1(t)$, $x_2(t)$, $x_3(t)$, $x_4(t)$ vs. time

$$
\begin{aligned}
D^{\alpha}\breve{a} &= -k_1 e_1((x_2 - x_1)) + \lambda_a e_a \\
D^{\alpha}\breve{b} &= -k_3 e_3 x_3 + \lambda_b e_b \\
D^{\alpha}\breve{c} &= -k_2 e_2 x_2 + \lambda_c e_c \\
D^{\alpha}\breve{d} &= -k_2 e_2 x_1 + \lambda_d e_d \\
D^{\alpha}\breve{r} &= -k_4 e_4 x_4 + \lambda_r e_r
\end{aligned} \tag{27}
$$

The simulation results of the predictive control combined with adaptive control for fractional order Rossler system and fractional order hyperchaotic Chen system are given.

C. Simulation Results of the Fractional Order Rossler System

The simulation results for the fractional order chaotic Rossler system are shown in this section. The simulation time is 100 seconds, and the controller is activated after 50 seconds. The time step size is 0.01 and the fractional order parameter is assumed to be 0.97. Fig. 2 depicts the results of the control of state signals x_1, x_2, and x_3. The parameter k is assumed to be -0.95.

It can be seen in Fig. 1 that after 50 seconds, $x_1(t)$ quickly reaches zero by applying the controller. Fig. 2 shows the estimation results of $\breve{a},\breve{b},\breve{c}$.

Fig. 3 shows that the parameters are well estimated from the start using Eq. (23) and without the control signal. And they changed from their original value at the 50s when the controller was applied, but they returned to their original value after the controller was removed.

D. Simulation Results of the Fractional Order Hyperchaotic Chen System

In this section, the results related to predictive control combined with adaptive control for Chen fractional order hyperchaotic system are shown. In this case, the simulation time is equal to 100 seconds,

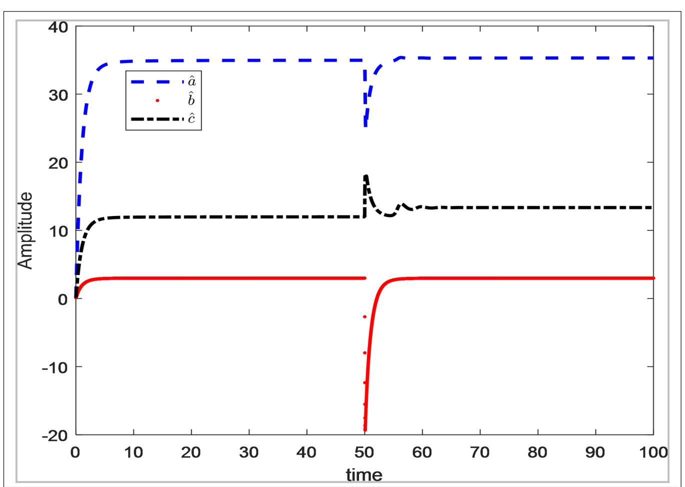

Fig. 5. Estimation of parameters $\breve{a},\breve{b},\breve{c}$ of the Chen fractional order hyperchaotic system vs. time

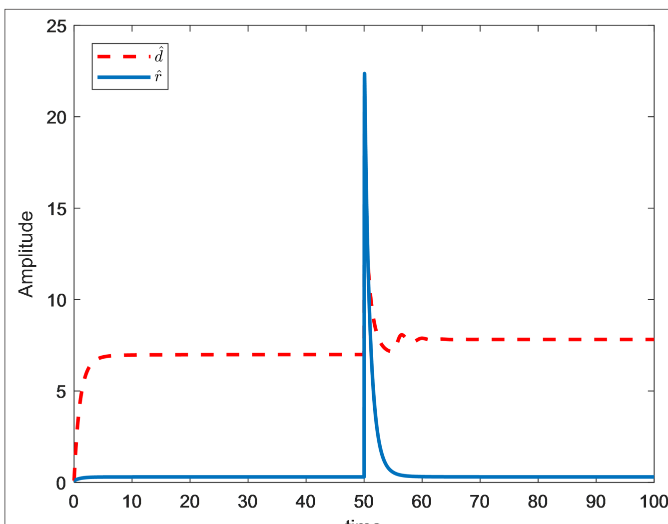

Fig. 6. Estimation of parameters $\breve{d},\breve{r}$ of the Chen fractional order hyperchaotic system vs. time

the time step size is 0.01, and the controller is applied at 50s. The results of applying the proposed controller are shown in Fig. 4 related to the states x_1,x_2,x_3,x_4. The controller gains k_1,k_2,k_3,k_4 were considered equal to -0.95. Control gains of $\lambda_a,\lambda_b,\lambda_c,\lambda_d,\lambda_r = -0.95$ are considered.

It can be seen from Fig. 4 that since the controller is applied, the system states have converged to the zero-equilibrium point. Fig. 5 and Fig. 6 show the parameter estimation results.

Figs. 5 and 6 show that the parameters are also estimated once the controller is applied. According to the results, the proposed design is capable of controlling fractional order Chen and Rossler systems and estimating their parameters.

V. CONCLUSION

The control problem of Rossler and Chen chaotic systems was investigated in this paper using the predictive feedback approach along with the estimation of unknown system parameters using adaptive control. The control of chaotic and hyperchaotic systems toward equilibrium points was researched using Lyapunov's stability theorem. The adaptive rules and controllers were then obtained with its assistance, allowing a set of fractional order chaotic systems to converge toward their equilibrium points. According to the results, all of the modes have converged toward the equilibrium point, and the parameters have also been appropriately determined. The practical application of the suggested adaptive predictive controller for synchronizing these systems while accounting for uncertainty and unknown parameters can be explored for future research in this subject. The practical application of the proposed controller can be used for Chua circuit. It is also proposed that the mentioned controller be used to control and synchronize chaotic and hyperchaotic systems with delayed fractional order.

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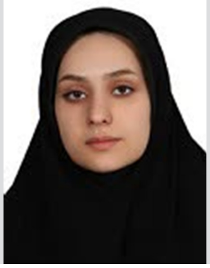

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