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Pattern Search Ameliorated Arithmetic Optimization Algorithm for Engineering Optimization and Infinite Impulse Response System Identification

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ABSTRACT

In the field of science and engineering, significant attention can be observed recently for system identification as a complex optimization problem. As the infinite impulse response (IIR) models can achieve more accurate models of physical plants for real-world applications, they are mostly preferred over finite impulse response models. Despite the latter advantage of the IIR structures, it is not straightforward to minimize the related cost functions as they tend to generate multimodal error surfaces. Metaheuristic algorithms have already been shown for their excellent promise to deal with such difficulties as they operate independent of the nature of the problem. In this regard, this work aims to demonstrate the excellent promise of a novel developed metaheuristic algorithm named pattern search ameliorated arithmetic optimization algorithm. The proposed algorithm integrates the original form of the arithmetic optimization algorithm (for exploration) with the pattern search algorithm (for exploitation) such that a better-performing metaheuristic structure is achieved. The excellent ability of the proposed pattern search ameliorated arithmetic optimization algorithm is demonstrated against the original arithmetic optimization algorithm by using well-known classical benchmark functions and welded beam design problem. A significant improvement is achieved for benchmark functions, and an improvement of up to 25% is obtained for the optimal cost of the welded beam design. Then, different IIR model identification problems are considered, and a comparative assessment is performed using different metaheuristic optimization techniques: particle swarm optimization algorithm, artificial bee colony algorithm, electromagnetism-like optimization algorithm, cuckoo search algorithm, and flower pollination algorithm. The obtained statistical results from different systems confirm that the pattern search ameliorated arithmetic optimization algorithm can achieve better accuracy and robustness in terms of IIR model identification.

Index Terms— Arithmetic optimization algorithm, digital IIR filters, engineering optimization, pattern search, swarm-based optimization, system identification.

I. INTRODUCTION

Researchers in the fields of science and engineering have recently paid great attention to one of the complex optimization problems known as system identification. The significance of system identification can be observed in the fields of parameter estimation [1], power systems [2], robotics [3], signal processing, communication, and control [4]. In system identification, an optimizer is used to minimize an error function (between the candidate model's output and the actual plant's output) for obtaining an optimal model for the unknown plant [5]. It is feasible to achieve an optimal model by effectively reducing the error function. Meanwhile, the sufficiency of the estimated model depends on the structure of the adaptive model and the characteristics of the input and output data as well as the optimizer.

Infinite impulse response (IIR) models can better represent the systems as they more accurately mimic the physical plants compared to their equivalent finite impulse response models [6]. Moreover, fewer parameters are required for IIR models in order to meet the performance specifications. Nevertheless, the structures of these models have difficult cost functions to minimize as they tend to generate errors with multimodal surfaces [7]. To deal with such difficulties, metaheuristic optimizers have recently been used as promising candidates, as they have been demonstrated to achieve better results in terms of accuracy and robustness [7–9]. In this regard, several metaheuristic algorithm examples for the IIR system identification problem can be found in the literature [10]. For example, a metaheuristic algorithm called average differential evolution with local search was proposed for identifying optimal coefficients of unknown IIR systems [11].

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By minimizing the error between the unknown system output and the adaptive IIR filter output, the proposed algorithm enables rapid convergence to global solutions in system identification problems, resulting in the precise prediction of filter coefficients on multimodal error surfaces. The performance of average differential evolution with local search algorithm was demonstrated through comparisons with other methods, showing its efficiency in terms of convergence rate and mean square error value. It is feasible to extend the examples such as firefly algorithm [4], teacher-learner-based optimization algorithm [6], whale optimization algorithm [7], selfish herd optimization algorithm, [8] and bat algorithm [12] as metaheuristic approaches reported for the IIR system identification problem.

This study aims to demonstrate the promise of another recently reported metaheuristic algorithm for IIR system identification by considering the fact that no optimizers can solve all existing problems competitively [13]. Therefore, in this work, the promise of the arithmetic optimization algorithm (AOA) [14] is comparatively presented for IIR model identification. The reason for the employment of the AOA arises from its demonstrated capability for several other applications [15–19]. This study also develops a novel metaheuristic algorithm as an excellent candidate for IIR model identification. The developed algorithm is an improved version of the AOA, which is constructed via appropriate integration of the pattern search algorithm [20].

In the proposed pattern search ameliorated AOA, the original AOA deals with the global search (exploration), while the pattern search algorithm takes care of the local search (exploitation). The more excellent ability of the proposed pattern search ameliorated AOA is demonstrated against the original AOA using 23 well-known classical benchmark functions. Here, the comparisons with the original AOA are evaluated to be sufficient, as the latter one has already been demonstrated for its promise comparatively against other available recent and well-performing metaheuristic algorithms [14]. Welded beam design problem is also adopted to further showcase the performance of the proposed methodology against the competitive approaches reported in the literature.

The abilities of the proposed pattern search ameliorated AOA and the original form of the AOA are then evaluated for IIR model identification. In this regard, a second-order plant with a first-order IIR model, a second-order plant with a second-order IIR model, and a high-order plant with a high-order IIR model are examined in order to demonstrate the more excellent promise of the proposed algorithm for IIR model identification. In terms of the comparisons, the popular optimizers of particle swarm optimization [21], artificial bee colony algorithm [22], and the electromagnetism-like optimization algorithm [23] together with other competitive optimizers of cuckoo search algorithm [24] and the flower pollination algorithm [25] are employed as the reported examples in the literature. The obtained results clearly demonstrate the excellent structure of the proposed pattern search ameliorated AOA for the IIR model identification. The key contributions of this work can be highlighted as follows:

- A novel algorithm that integrates the global search capabilities of the AOA with the local search capabilities of the pattern search algorithm is described.
- The proposed pattern search ameliorated AOA is evaluated against benchmark functions and its enhanced ability compared to the original AOA.
- Superiority of the proposed approach is further demonstrated using welded beam design problem as a real-world engineering optimization in order to further showcasing the performance of the proposed algorithm.
- The promise of the pattern search ameliorated AOA is presented for IIR model identification using various plant and model configurations through rigorous evaluations and comparisons with popular optimizers.

The remainder of this paper is organized as follows: Section 2 delves into the structure of the proposed pattern search ameliorated AOA, which is comprised of three key components: the AOA (Section 2.1), the pattern search algorithm (Section 2.2), and the proposed pattern search ameliorated AOA (Section 2.3). In Section 3, we conduct a thorough performance assessment of the pattern search ameliorated AOA against classical benchmark functions and welded beam design problem. Section 4 focuses on the application of the algorithm to IIR system identification and filter design. Moving on to Section 5, we present the simulation results obtained and engage in insightful discussions. Finally, in Section 6, we draw conclusions based on our findings and discuss potential future directions for research in this field.

II. THE STRUCTURE OF THE PROPOSED PATTERN SEARCH AMELIORATED ARITHMETIC OPTIMIZATION ALGORITHM

A. Arithmetic Optimization Algorithm

The AOA uses the arithmetic operators to construct a metaheuristic optimizer [14]. It starts with the generation of a matrix that consists of a set of random solutions. Then, the exploration and exploitation tasks are performed based on the following function:

$$MOA(t) = Min + t_c \times \left(\frac{Max - Min}{t_{max}} \right) \tag{1}$$

In here, $MOA(t)$ represents the current iteration's function value, t_c denotes the current iteration, and t_{max} is the maximum number of iterations, whereas *Min* and *Max* are respectively the minimum and maximum values of the *MOA (math optimizer accelerated)* function. The exploration is performed for $r_1 > MOA$ where r_1 is a random number. In this stage, the multiplication (*Mult*) and division (*Div*) operators are used, which are defined in (2).

$$x_{i,j}(t_c+1) = \begin{cases} best(x_j) \times MOP \times \left((UB_j - LB_j) \times \mu + LB_j \right), for\ r_2 > 0.5 \\ best(x_j) \div (MOP + \varepsilon) \times \left((UB_j - LB_j) \times \mu + LB_j \right), for\ r_2 < 0.5 \end{cases} \tag{2}$$

Here, the *j*th position of solution *i* for current iteration is represented by $x_{i,j}(t_c)$, the solution of *i* in the next iteration is denoted by $x_i(t_c+1)$, the best solution's *j*th position (obtained so far) is shown by *best* (x_j), ● is a small integer number, and μ is a control parameter adjusting the search process, whereas UB_j and LB_j are respectively the upper and lower bounds of the *j*th position. r_2 represents another random number used for the position update. The *Mult* operator is used for r_2 > 0.5, otherwise the *Div* operator is employed. The *MOP* function is calculated as given in (3).

$$MOP(t_c) = 1 - \frac{(t_c)^{1/\alpha}}{(t_{max})^{1/\alpha}} \tag{3}$$

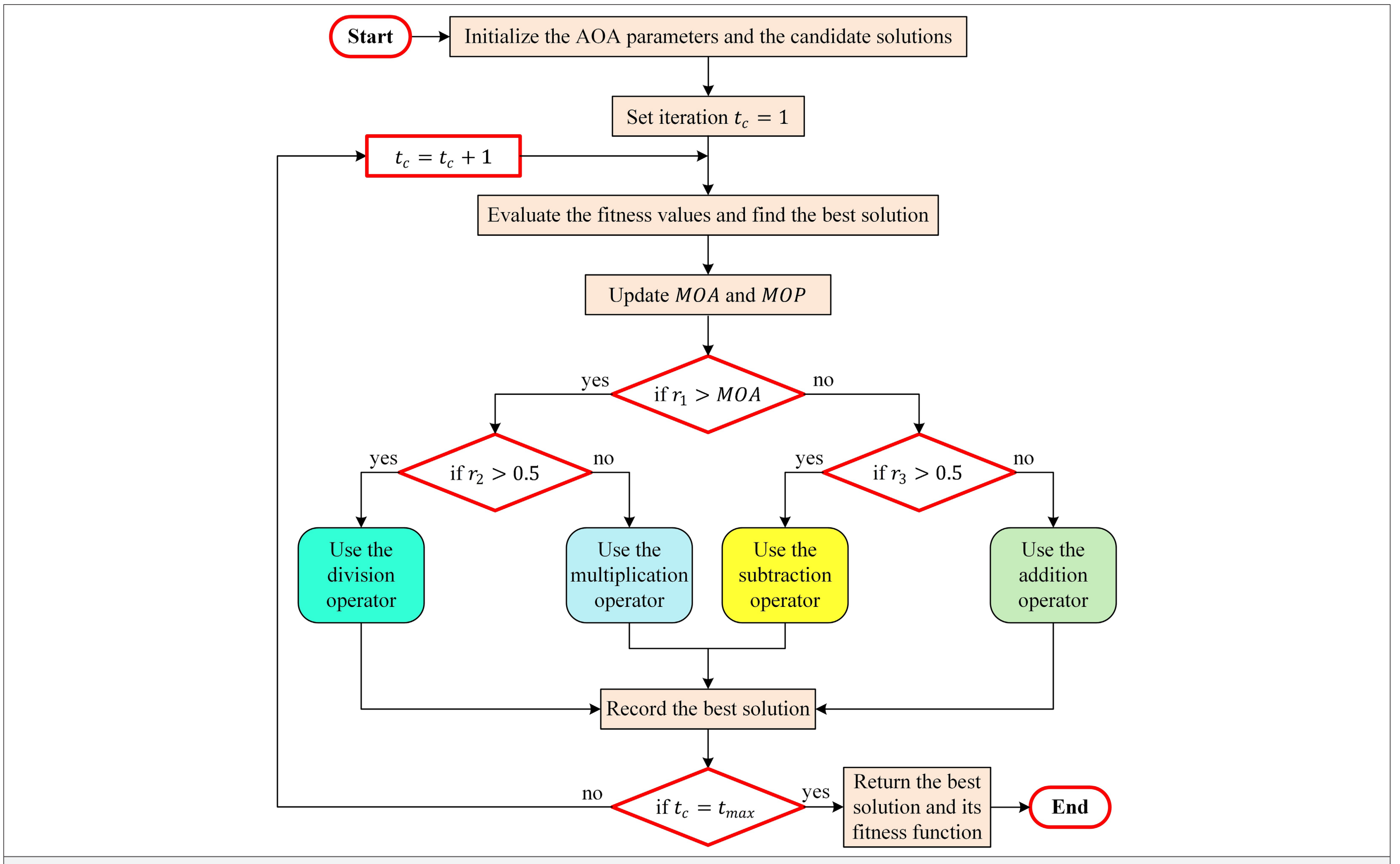

Fig. 1. Flowchart for arithmetic optimization algorithm.

where α is the exploitation accuracy. For $r_1 < MOA$, the exploitation stage takes place where the addition (*Add*) and subtraction (*Sub*) operators are employed using the following definition: The *Add* operator performs for $r_3 > 0.5$, and the *Sub* is used for $r_3 < 0.5$, where r_3 stands for a random number. The flowchart of the AOA is provided in Fig. 1.

$$x_{i,j}(t_c+1) = \begin{cases} best(x_j) + MOP \times ((UB_j - LB_j) \times \mu + LB_j), for\ r_3 > 0.5 \\ best(x_j) - MOP \times ((UB_j - LB_j) \times \mu + LB_j), for\ r_3 < 0.5 \end{cases} \quad (4)$$

B. Pattern Search Algorithm

The pattern search algorithm has a derivative-free mechanism that allows to reach better exploitation ability [20]. This algorithm starts to perform by generating a point that may or may not be close to the solution [26]. Around the generated point, the pattern search algorithm creates a collection of points named mesh, which is updated if a new point with a lower objective function value in the mesh is found in the next iteration of the algorithm [27]. A starting point, X_0, for the search is defined by the user, and the size of the mesh is considered as 1 in the first iteration. This is followed by constructing the pattern vectors as $X_0+[0\ 1]$, $X_0+[1\ 0]$, $X_0+[-1\ 0]$, and $X_0+[0\ -1]$ to produce the new mesh points. The calculation of the objective functions of these points continues until a smaller value than X_0 is found. The source point is relocated in case of finding a smaller value, e.g., $f(X_1) < f(X_0)$, which is called successful poll. The following iteration of the pattern search algorithm (expanding stage) is performed by multiplying the current mesh size by 2 after successful poll [28]. That means the following new points are created as $X_1+2 \times [0\ 1]$, $X_1+2 \times [1\ 0]$, $X_1+2 \times [-1\ 0]$ and $X_1+2 \times [0\ -1]$. The latter stage continues until a newer point with lower objective function is found; otherwise, the mesh size is reduced by multiplying it with 0.5 (reduction factor). This is called the contracting stage. The overall process continues until the termination condition is met. Fig. 2 illustrates the representation of the mesh points and the direction in the pattern search algorithm.

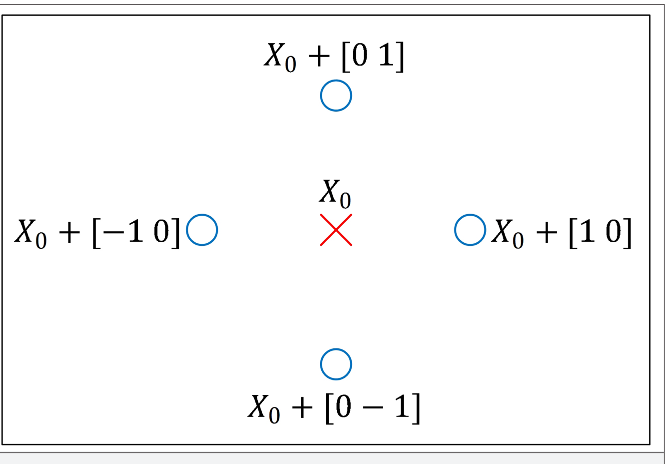

Fig. 2. Pattern search mesh points and the pattern.

C. The Proposed Pattern Search Ameliorated Arithmetic Optimization Algorithm

The proposed pattern search ameliorated AOA was constructed with the aim of reaching better exploration (global search) and exploitation (local search) capabilities. In this regard, the original structure of AOA was employed for exploration while the pattern search algorithm was integrated for exploitation. The proposed pattern search ameliorated AOA starts its operation with the original form of the AOA initially for the purpose of exploration. Then, the pattern search algorithm is run for the exploitation. However, the pattern search algorithm is called after every ten iterations throughout the process and for each call it runs for total number of iterations. This latter procedure was determined in order to reach a good balance between the exploration and exploitation stages. The overall procedure is repeated until the termination condition (total number of iterations) is satisfied. Fig. 3 provides a detailed flowchart of the proposed pattern search ameliorated AOA.

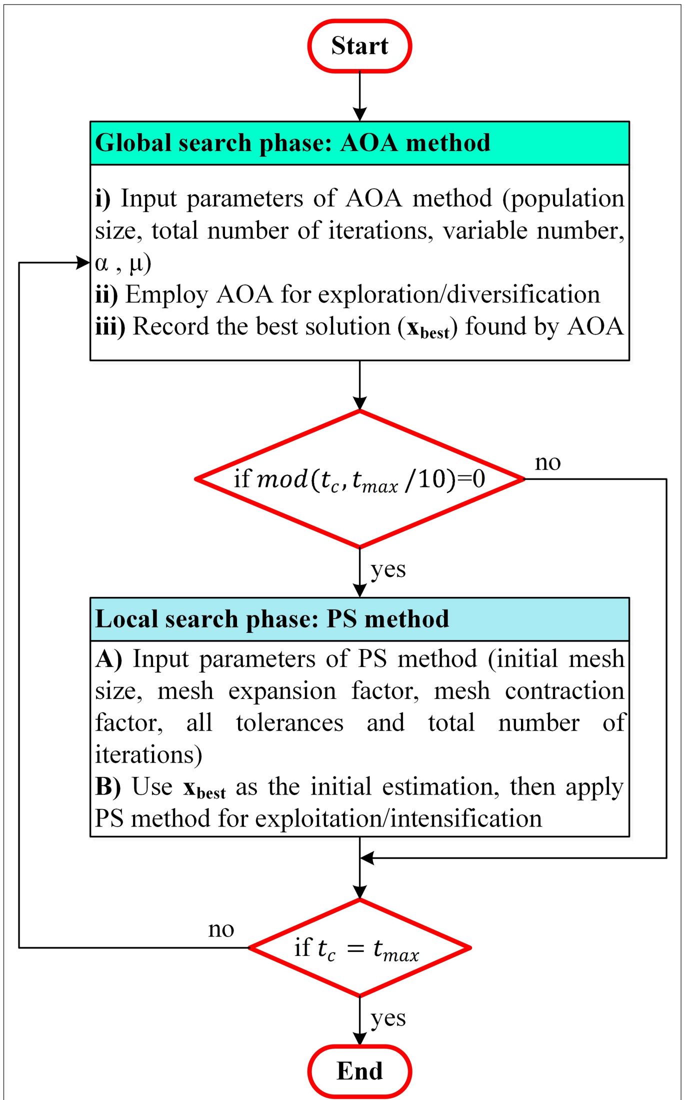

Fig. 3. Flowchart of proposed pattern search ameliorated arithmetic optimization algorithm.

III. PERFORMANCE ASSESSMENT OF THE PATTERN SEARCH AMELIORATED ARITHMETIC OPTIMIZATION ALGORITHM

The performance assessment of the constructed pattern search ameliorated AOA was initially carried out using the benchmark functions listed in Table I [29]. For the evaluations, the following parameters were used for the pattern search and the AOAs.

Pattern Search Algorithm: *initial mesh size* = 1, *mesh expansion factor* = 2, *mesh contraction factor* = 0.5, *all tolerances* = 10^{-6}.

Arithmetic Optimization Algorithm: sensitive parameter $\alpha = 5$, control parameter $\mu = 0.4975$, Min = 0.2, *Max* = 1.

Apart from the above-listed parameters regarding the algorithms, all algorithms were run 30 times with a maximum iteration $t_{max} = 500$ and population size $N = 30$ for the optimization of all test functions.

The obtained results for the used test functions are provided in Table II. It is worth noting that the comparisons were only performed

TABLE I. EMPLOYED BENCHMARK FUNCTIONS

ID	Name	Dimension	Search Range	F_{min}
F1	Sphere	30	[−100, 100]	0
F2	Schwefel 2.2	30	[−10, 10]	0
F3	Schwefel 1.2	30	[−100, 100]	0
F4	Schwefel 2.21	30	[−100, 100]	0
F5	Rosenbrock	30	[−30, 30]	0
F6	Step	30	[−100, 100]	0
F7	Quartic	30	[−1.28, 1.28]	0
F8	Schwefel	30	[−500, 500]	−1.2569E+04
F9	Rastrigin	30	[−5.12, 5.12]	0
F10	Ackley	30	[−32, 32]	0
F11	Griewank	30	[−600, 600]	0
F12	Penalized	30	[−50, 50]	0
F13	Penalized2	30	[−50, 50]	0
F14	Foxholes	2	[−65.536, 65.536]	0.998
F15	Kowalik	4	[−5, 5]	3.0749E−04
F16	Six-hump camel	2	[−5, 5]	−1.0316
F17	Branin	2	[−5, 10] × [0, 15]	0.39789
F18	Goldstein-price	2	[−2, 2]	3
F19	Hartman	3	[0, 1]	−3.8628
F20	Hartman 6	6	[0, 1]	−3.322
F21	Shekel 5	4	[0, 10]	−10.1532
F22	Shekel 7	4	[0, 10]	−10.4029
F23	Shekel 10	4	[0, 10]	−10.5364

TABLE II. STATISTICAL RESULTS FOR THE USED BENCHARK FUNCTIONS

ID	Algorithm	Average	Standard Deviation	Minimum	Maximum
F1	Arithmetic optimization algorithm	5.5888E−03	1.3545E−02	7.0990E−62	6.2354E−02
	Pattern search ameliorated arithmetic optimization algorithm	7.2395E−26	4.6741E−26	0	1.7879E−25
F2	Arithmetic optimization algorithm	1.0078E−166	0	3.6927E−293	3.0234E−165
	Pattern search ameliorated arithmetic optimization algorithm	2.9146E−211	0	1.2603E−304	8.6877E−210
F3	Arithmetic optimization algorithm	9.4005E−01	1.2370E+00	2.2040E−04	4.8821E+00
	Pattern search ameliorated arithmetic optimization algorithm	4.3903E−01	5.5961E−01	6.9546E−13	2.4339E+00
F4	Arithmetic optimization algorithm	1.6322E−01	9.0308E−02	4.2518E−03	3.8891E−01
	Pattern search ameliorated arithmetic optimization algorithm	2.2118E−14	4.0884E−14	2.5535E−15	2.2027E−13
F5	Arithmetic optimization algorithm	2.8656E+01	3.0276E−01	2.8088E+01	2.9022E+01
	Pattern search ameliorated arithmetic optimization algorithm	2.3515E+00	5.3589E−01	9.1267E−01	3.4357E+00
F6	Arithmetic optimization algorithm	3.7921E+00	2.9646E−01	3.1680E+00	4.5528E+00
	Pattern search ameliorated arithmetic optimization algorithm	1.1634E−12	3.2163E−13	5.9145E−13	2.1522E−12
F7	Arithmetic optimization algorithm	9.7396E−05	8.9538E−05	7.5463E−06	3.4752E−04
	Pattern search ameliorated arithmetic optimization algorithm	3.7979E−05	3.1375E−05	9.5038E−07	1.2071E−04
F8	Arithmetic optimization algorithm	−8.1498E+03	5.2109E+02	−9.1468E+03	−6.8502E+03
	Pattern searchameliorated arithmetic optimization algorithm	−8.1852E+03	5.0727E+02	−9.2920E+03	−7.3668E+03
F9	Arithmetic optimization algorithm	0	0	0	0
	Pattern search ameliorated arithmetic optimization algorithm	0	0	0	0
F10	Arithmetic optimization algorithm	8.8818E−16	0	8.8818E−16	8.8818E−16
	Pattern search ameliorated arithmetic optimization algorithm	8.8818E−16	0	8.8818E−16	8.8818E−16
F11	Arithmetic optimization algorithm	1.9237E+02	5.5001E+01	3.2363E+01	3.0140E+02
	Pattern search ameliorated arithmetic optimization algorithm	9.0186E−15	4.5979E−14	0	2.5235E−13
F12	Arithmetic optimization algorithm	2.8641E−01	4.3314E−02	2.1061E−01	4.1247E−01
	Pattern search ameliorated arithmetic optimization algorithm	1.9003E−13	1.5979E−13	1.8251E−14	5.8831E−13
F13	Arithmetic optimization algorithm	2.5472E+00	1.6996E−01	2.1305E+00	2.8367E+00
	Pattern search ameliorated arithmetic optimization algorithm	2.0724E−02	4.7140E−02	1.0411E−13	2.0724E−01
F14	Arithmetic optimization algorithm	6.7934E+00	2.5465E+00	1.9920E+00	1.2671E+01
	Pattern search ameliorated arithmetic optimization algorithm	2.0458E+00	2.5010E+00	9.9800E−01	1.0763E+01
F15	Arithmetic optimization algorithm	2.0470E−02	2.9965E−02	3.5276E−04	1.0144E−01
	Pattern search ameliorated arithmetic optimization algorithm	6.2146E−04	9.4035E−04	3.0749E−04	5.4515E−03
F16	Arithmetic optimization algorithm	−1.0316E+00	0	−1.0316E+00	−1.0316E+00
	Pattern search ameliorated arithmetic optimization algorithm	−1.0316E+00	0	−1.0316E+00	−1.0316E+00
F17	Arithmetic optimization algorithm	4.1299E−01	1.1040E−02	3.9854E−01	4.4280E−01
	Pattern search ameliorated arithmetic optimization algorithm	3.9789E−01	0	3.9789E−01	3.9789E−01
F18	Arithmetic optimization algorithm	1.2176E+01	1.5334E+01	3	6.2268E+01
	Pattern search ameliorated arithmetic optimization algorithm	3	0	3	3

(*Continued*)

TABLE II. STATISTICAL RESULTS FOR THE USED BENCHARK FUNCTIONS (*CONTINUED*)

ID	Algorithm	Average	Standard Deviation	Minimum	Maximum
F19	Arithmetic optimization algorithm	−3.8511E+00	4.3328E−03	−3.8585E+00	−3.8382E+00
	Pattern search ameliorated arithmetic optimization algorithm	−3.8628E+00	0	−3.8628E+00	−3.8628E+00
F20	Arithmetic optimization algorithm	−3.0158E+00	1.2254E−01	−3.2440E+00	−2.6474E+00
	Pattern search ameliorated arithmetic optimization algorithm	−3.2943E+00	5.1149E−02	−3.3220E+00	−3.2031E+00
F21	Arithmetic optimization algorithm	−3.7069E+00	1.3780E+00	−6.9692E+00	−1.4605E+00
	Pattern search ameliorated arithmetic optimization algorithm	−8.4786E+00	3.1151E+00	−1.0153E+01	−2.6305E+00
F22	Arithmetic optimization algorithm	−3.3648E+00	1.0538E+00	−5.1110E+00	−1.3028E+00
	Pattern search ameliorated arithmetic optimization algorithm	−8.6984E+00	3.1679E+00	−1.0403E+01	−2.7659E+00
F23	Arithmetic optimization algorithm	−4.1793E+00	1.3834E+00	−7.8578E+00	−1.5451E+00
	Pattern search ameliorated arithmetic optimization algorithm	−1.0021E+01	1.9610E+00	−1.0536e+01	−2.8066E+00

against the original AOA, as it has already been demonstrated to be more excellent compared to other recent and capable metaheuristic algorithms [14]. The presented results in Table II, on the other hand, shows more excellent capability of the proposed pattern search ameliorated AOA proposed in this study, as it reaches far better or optimum results for the used test functions.

For further assessment and performance showcase, the proposed pattern search ameliorated AOA was also used to solve a real-world engineering optimization problem known as welded beam design problem. The main objective of this problem is to find the minimum fabrication cost by defining the optimal value of the given variables, which are four optimization variables named length of attached part of bar (*l*), thickness of weld (*h*), the height of the bar (*t*), and thickness of the bar (*b*). The given variables need to be satisfied with relevant constraints. As the objective is to minimize the fabrication cost, the cost function can be defined as $cost = w1 \times (A \times l + B \times h + C \times t + D \times b)$, which is subject to the constraints of $l >= l\_min$; $l <= l\_max$; $h >= h\_min$; $h <= h\_max$; $t >= t\_min$; $t <= t\_max$; $b >= b\_min$ and $b <= b\_max$ where *A*, *B*, *C*, *D* are weights associated with the lengths and thicknesses, respectively; *l_min*, *l_max* are minimum and maximum allowable values for *l*; *h_min*, *h_max* are minimum and maximum allowable values for *h*; *t_min*, *t_max* are minimum and maximum allowable values for *t* and *b_min*, *b_max* are minimum and maximum allowable values for *b*. In this formulation, the cost function represents the fabrication cost of the welded beam design, considering the lengths and thicknesses of different components. The objective is to minimize this cost while satisfying the given constraints related to the values of *l*, *h*, *t*, and *b* within their respective allowable ranges.

By utilizing the cost function and metaheuristic algorithms, the design space can effectively be explored, promising solutions can be identified, and ultimately converge towards an optimal design that meets the desired cost criteria while satisfying the specified constraints can be achieved. The proposed pattern search ameliorated AOA was applied for solving the welded beam design and compared with several optimization algorithms published in the literature (genetic algorithm [30], harmony search algorithm [31], whale optimization algorithm [32], gravitational search algorithm [33], multiverse optimizer [33] and the original AOA [14]). Table III provides the adopted algorithms for comparisons and the respective optimal costs achieved by them. As can be concluded from the presented results, the proposed pattern search ameliorated AOA is capable of reaching the best optimal cost (highlighted in bold), declaring its efficacy for real-world engineering design problems.

TABLE III. RESULTS OF THE COMPARATIVE ALGORITHMS FOR SOLVING THE WELDED BEAM DESIGN PROBLEM

	Optimal Values for Variables				
Algorithm	***H***	***l***	***t***	***b***	**Optimal Cost**
Genetic algorithm	0.2489	6.1730	8.1789	0.2533	2.4300
Harmony search algorithm	0.2442	6.2231	8.2915	0.2400	2.3807
Whale optimization algorithm	0.205396	3.484293	9.037426	0.206276	1.730499
Gravitational search algorithm	0.182129	3.856979	10.000	0.202376	1.87995
Multi-verse optimizer	0.205463	3.473193	9.044502	0.205695	1.72645
Arithmetic optimization algorithm	0.194475	2.57092	10.000	0.201827	1.7164
Pattern search ameliorated arithmetic optimization algorithm (proposed)	0.203202	3.300884	9.028726	0.206090	**1.699337**

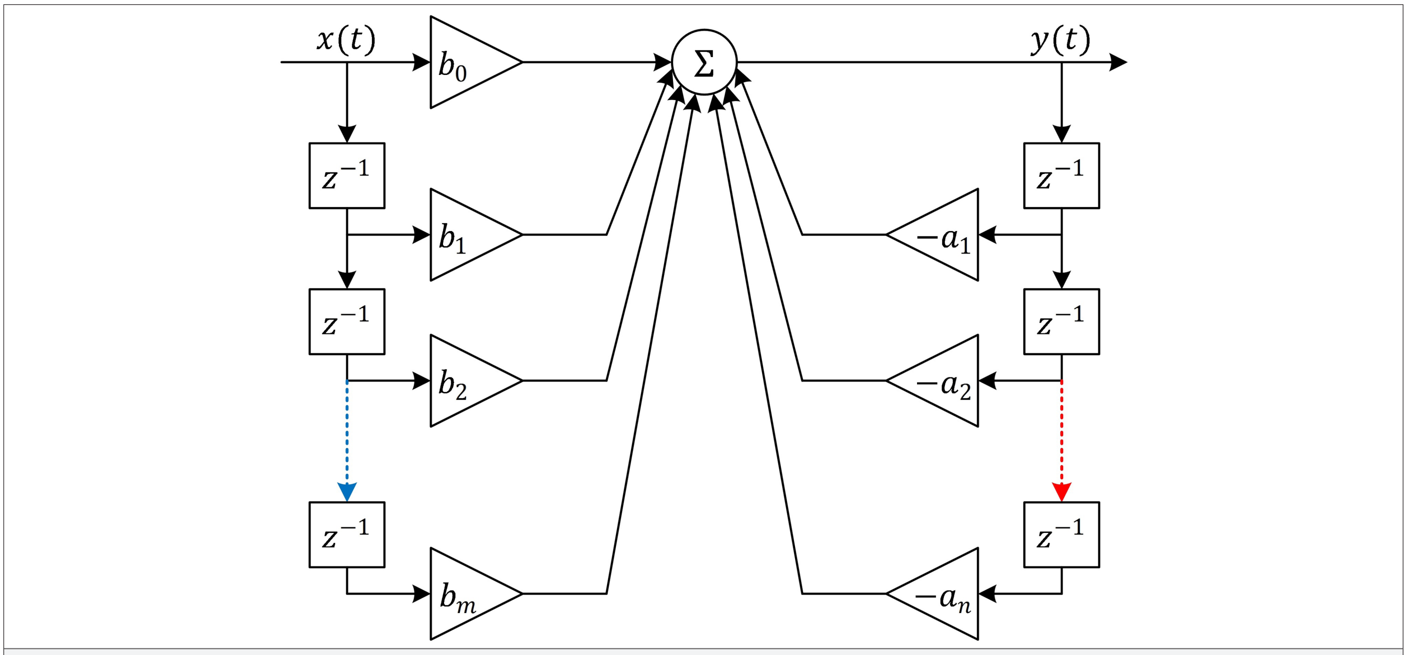

Fig. 4. Infinite impulse response filter structure.

IV. INFINITE IMPULSE RESPONSE SYSTEM IDENTIFICATION AND FILTER DESIGN

System identification refers to the representation of an unknown system mathematically by considering input and output data. An optimization algorithm is used to minimize an error function (between the candidate model's output and the actual plant's output) in order to obtain an optimal model for the unknown plant. On the other hand, fewer model parameters can be used via IIR models to meet the performance specifications and produce a more accurate representation of physical plants for real-world applications [25]. An arbitrary system's IIR identification model is illustrated in Fig. 4 where $y(t)$ and $d(t)$, respectively, represent the output of the IIR filter and the unknown plant. On the other hand, $x(t)$ stands for the applied input signal, whereas m and n are, respectively, the coefficients of the numerator and denominator.

In the light of the information provided in Fig. 4, the following form represents the transfer function of an IIR system.

$$\frac{Y(z)}{X(z)}=\frac{b_0+b_1z^{-1}+b_2z^{-2}+\ldots+b_mz^{-m}}{1+a_1z^{-1}+a_2z^{-2}+\ldots+a_nz^{-n}} \tag{5}$$

Here, the pole and zero parameters of the IIR model are denoted by a_i and b_j where $i=1, 2, \ldots, n$ and $j=0, 1, \ldots, m$. The difference equation form of the transfer function given in (5) can be written as:

$$y(t)+\sum_{i=1}^{n}a_i\cdot y(t-i)=\sum_{j=0}^{m}b_j\cdot x(t-j) \tag{6}$$

where x (t) and $y(t)$ represent the input and the output of the filter, respectively. Fig. 5 demonstrates the block diagram of an adaptive IIR filter designed via the arithmetic optimization and the proposed pattern search ameliorated AOA for the system identification purpose.

In here, $e(t)$ represents the error between the model and the actual plant as $e(t)=d(t)-y(t)$, which can be used for considering the infinite impulse response model identification problem as a minimization problem described by the following function.

$$f(\theta)=\frac{1}{W}\sum_{t=1}^{W}\left(d(t)-y(t)\right)^2 \tag{7}$$

In here, W represents the number of samples employed in the simulation. With the adjustment of θ, it is aimed to minimize the $f(\theta)$ cost function.

V. SIMULATION RESULTS AND DISCUSSIONS

In this work, three experiments were considered. In the first experiment, a second-order plant with a first order IIR model was examined,

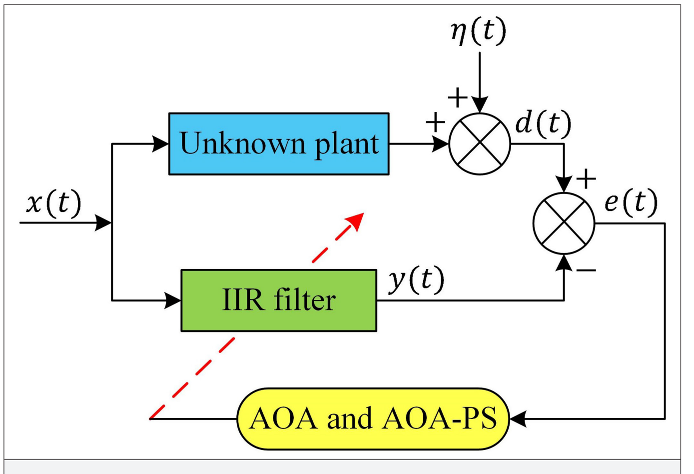

Fig. 5. Block diagram of adaptive infinite impulse response filter designed via AOA and AOA-PS algorithms for system identification.

TABLE IV. PARAMETER SETTINGS OF THE ADOPTED ALGORITHMS FOR COMPARISONS

Algorithm	Total Iteration Number	Population Size	Other Parameters
Pattern search ameliorated arithmetic optimization algorithm (proposed)	500	25	$\alpha = 5$, $\mu = 0.4975$, *initial mesh size* = 1, *mesh expansion factor* = 2, *mesh contraction factor* = 0.5, *all tolerances* = 10^{-6}
Arithmetic optimization algorithm (proposed)	500	25	$\alpha = 5$, $\mu = 0.4975$
Particle swarm optimization	3000	25	$C_1 = 2$, $c_2 = 2$, weight factor decreases linearly from 0.9 to 0.2
Artificial bee colony algorithm	3000	25	*limit* = 100
Electromagnetism-like optimization algorithm	3000	25	$\delta = 0.001$, *LSITER* = 4
Cuckoo search algorithm	3000	25	$p_a = 0.25$,
Flower pollination algorithm	3000	25	$p = 0.8$

whereas in the second and third experiments a second-order plant with a second order IIR model and a high-order plant with a high-order model were respectively considered. In this work, the obtained results with the original form of the AOA together with the proposed pattern search ameliorated AOA were compared with the reported works based on particle swarm optimization, artificial bee colony algorithm, electromagnetism-like optimization algorithm, cuckoo search algorithm, and flower pollination algorithm. In the experiments, the parameters of the algorithm were set as presented in Table IV. Besides, for all simulations, input signal ($x(t)$) is white noise sequence (zero mean, variance 0.1) with $W = 100$. As can be observed from the table, the proposed pattern search ameliorated AOA performed with the lower total number of iterations. Nonetheless, it has found good solutions, as will be observed from the experimental results that are presented in the following subsections, indicating its superior performance for versatile complex problems.

A. First Experiment

In the first experiment, it was aimed to identify a second-order plant through a first-order IIR model. The transfer functions given in eqs. (8) and (9) are used for the unknown plant (H_P) and the IIR model (H_M) for such a case.

$$H_P\left(z^{-1}\right) = \frac{0.05 - 0.4z^{-1}}{1 - 1.1314z^{-1} + 0.25z^{-2}} \quad (8)$$

$$H_M\left(z^{-1}\right) = \frac{b}{1 - az^{-1}} \quad (9)$$

Table V reports the performance assessment (the best parameter values of a and b, the average value and the standard deviation of $f(\theta)$ cost function) results obtained after 50 executions.

As demonstrated in Table V, the proposed pattern search ameliorated AOA achieved the best results for the average and standard deviation values compared to the algorithms of the original AOA, particle swarm optimization, artificial bee colony algorithm, electromagnetism-like optimization algorithm, cuckoo search algorithm, and flower pollination algorithm. Therefore, the proposed pattern search ameliorated AOA is able to maintain a significant precision and robustness due to the lowest average and standard deviation values of $f(\theta)$ cost function. This can also be observed from the convergence curve provided in Fig. 6.

B. Second Experiment

In the second experiment, it was aimed to identify a second-order plant through a second-order IIR model. The transfer functions given in eqs. (10) and (11) are used for the unknown plant (H_P) and the IIR model (H_M) for the second experiment.

TABLE V. FIRST EXPERIMENT'S PERFORMANCE RESULTS

	Parameter		Statistical Metric	
Algorithm	*a*	*b*	Average Value	Standard Deviation
Pattern search ameliorated arithmetic optimization algorithm (proposed)	0.9014	−0.3364	**9.6212E−03**	**1.0777E−04**
Arithmetic optimization algorithm (proposed)	0.8991	−0.3246	9.8775E−03	1.2584E−04
Particle swarm optimization	0.9125	−0.3012	0.0284	0.0105
Artificial bee colony algorithm	0.1420	−0.3525	0.0197	0.0015
Electromagnetism-like optimization algorithm	0.9034	0.3030	0.0165	0.0012
Cuckoo search algorithm	0.9173	−0.2382	0.0101	3.118E−04
Flower pollination algorithm	0.9364	−0.2001	0.0105	5.103E−04

Bold values are signifying the best values.

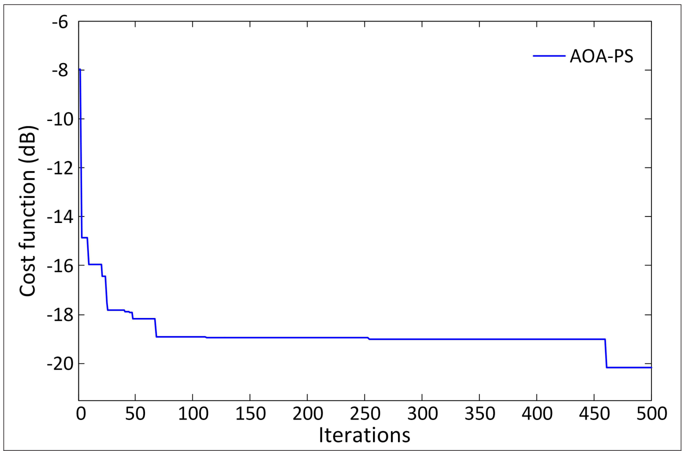

Fig. 6. Convergence curve of the pattern search ameliorated arithmetic optimization algorithm for the first experiment.

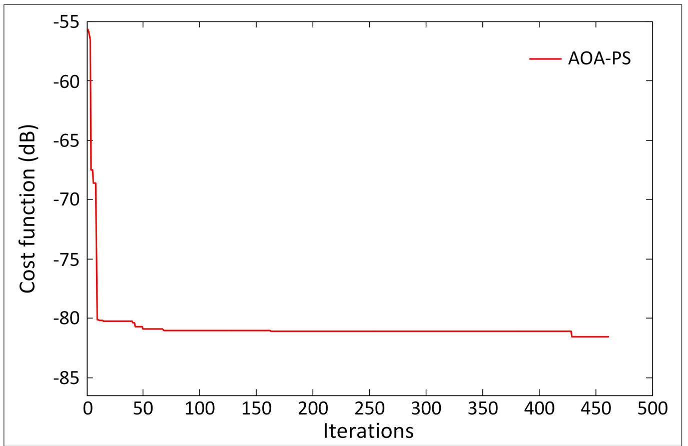

Fig. 7. Convergence curve of the pattern search ameliorated arithmetic optimization algorithm for the second experiment.

$$H_P\left(z^{-1}\right)=\frac{1}{1-1.4z^{-1}+0.49z^{-2}} \quad (10)$$

$$H_M\left(z^{-1}\right)=\frac{b}{1+a_1z^{-1}+a_2z^{-2}} \quad (11)$$

Similar to the first experiment, the results for this experiment were obtained after 50 executions. The respective numerical values are reported in Table VI. The related results show that the original AOA, cuckoo search algorithm, and the proposed pattern search ameliorated AOA perform better compared to particle swarm optimization, artificial bee colony algorithm, electromagnetism-like optimization algorithm, and flower pollination algorithm, demonstrating highly competitive performance of the proposed pattern search ameliorated AOA. This can also be observed from the convergence curve provided in Fig. 7.

C. Third Experiment

In the last experiment, it was aimed to identify a superior-order plant through a high-order IIR model. The transfer functions given in eqs. (12) and (13) are used for the unknown plant (H_P) and the IIR model (H_M) for the last experiment.

$$H_P\left(z^{-1}\right)=\frac{1-0.4z^{-2}-0.65z^{-4}+0.26z^{-6}}{1-0.77z^{-2}-0.8498z^{-4}+0.6486z^{-6}} \quad (12)$$

$$H_M\left(z^{-1}\right)=\frac{b_0+b_1z^{-1}+b_2z^{-2}+b_3z^{-3}+b_4z^{-4}}{1+a_1z^{-1}+a_2z^{-2}+a_3z^{-3}+a_4z^{-4}} \quad (13)$$

Table VII lists the comparative best parameters obtained and the statistical metrics for cost function. As seen in the respective table, the proposed pattern search ameliorated AOA achieves far better results compared to original AOA, particle swarm optimization, artificial bee colony algorithm, electromagnetism-like optimization algorithm, cuckoo search algorithm, and flower pollination algorithm, demonstrating its significant capability to present better precision and robustness for such a higher-order model. This can also be observed from the convergence curve provided in Fig. 8.

D. Effect of Measurement Noise on Performance of the Proposed Algorithm

The last assessment of the pattern search ameliorated AOA-based IIR system identification is performed to observe the effect of the measurement noise on its performance for the first, second, and

TABLE VI. SECOND EXPERIMENT'S PERFORMANCE RESULTS

	Parameter			Statistical Metric	
Algorithm	a_1	a_2	b	**Average Value**	**Standard Deviation**
Pattern search ameliorated arithmetic optimization algorithm (proposed)	−1.4000	0.4900	1.0000	**0.0000**	**0.0000**
Arithmetic optimization algorithm (proposed)	−1.4000	0.4900	1.0000	**0.0000**	**0.0000**
Particle swarm optimization	−1.4024	0.4925	0.9706	4.0035E−05	1.3970E−05
Artificial bee colony algorithm	−1.2138	0.6850	0.2736	0.3584	0.1987
Electromagnetism-like optimization algorithm	−1.0301	0.4802	1.0091	3.9648E−05	8.7077E−05
Cuckoo search algorithm	−1.4000	0.4900	1.0000	**0.0000**	**0.0000**
Flower pollination algorithm	−1.4000	0.4900	1.0000	4.6246E−32	2.7360E−31

The bold values are signifying the best values.

TABLE VII. THIRD EXPERIMENT'S PERFORMANCE RESULTS

	Parameter									Statistical Metric	
Algorithm	a_1	a_2	a_3	a_4	b_0	b_1	b_2	b_3	b_4	**Average Value**	**Standard Deviation**
Pattern search ameliorated arithmetic optimization algorithm (proposed)	−0.0033	0.0062	0.0024	−0.8595	0.9991	−0.0183	0.3675	−0.0078	−0.3737	**4.4689E−05**	**9.0969E−07**
Arithmetic optimization algorithm (proposed)	−0.0326	0.0220	0.0478	−0.8725	0.9945	−0.0417	0.4048	0.0578	−0.3930	2.3952E−04	4.7217E−06
Particle swarm optimization	0.3683	−0.7043	0.2807	0.3818	0.9939	−0.6601	−0.8520	0.2275	−1.4990	5.8843	3.4812
Artificial bee colony algorithm	−1.1634	−0.6354	−1.5182	0.6923	0.5214	−1.2703	0.3520	1.1816	−1.9411	7.3067	4.3194
Electromagnetism-like optimization algorithm	−0.4950	−0.7049	0.5656	−0.2691	1.0335	−0.6670	−0.4682	0.6961	−0.0673	0.0140	0.0064
Cuckoo search algorithm	0.9599	0.0248	0.0368	−0.0002	−0.2377	0.0031	−0.3579	0.0011	−0.5330	6.7515E−04	4.1451E−04
Flower pollination algorithm	0.0328	−0.1059	−0.0243	−0.7619	1.0171	0.0038	0.2374	0.0259	−0.3365	0.0018	0.0020

Bold values are signifying the best values.

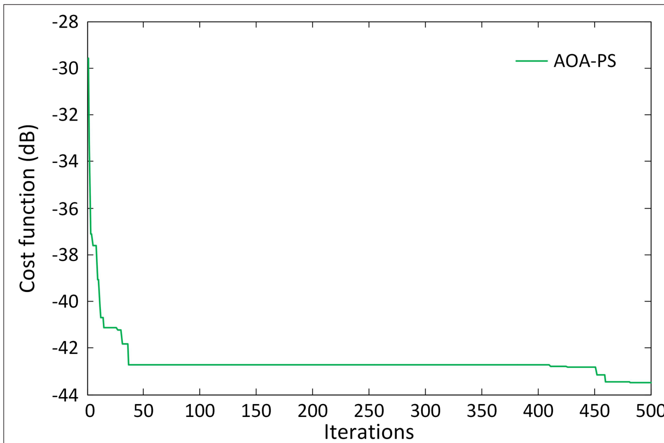

Fig. 8. Convergence curve of the pattern search ameliorated arithmetic optimization algorithm for the third experiment.

third experiments. The measurement noise for the implementation is taken as a Gaussian white signal with variance 10^{-3}. Table VIII provides the statistical results of $f(\theta)$ cost function values for the first, second, and third experiments (with and without noise). Looking at the numerical results of the statistical metrics provided in Table VIII, one can observe that there is no significant difference between all the systems with and without noise. As the proposed pattern search ameliorated AOA is not affected significantly by the measurement noise, it is robust against disturbance effects.

TABLE VIII. STATISTICAL RESULTS OF COST FUNCTION VALUES FOR THE FIRST, SECOND AND THIRD EXPERIMENTS (WITH AND WITHOUT NOISE)

Test system	Metric	Without Noise	With Noise
First experiment	Average	9.6212E−03	9.9799E−03
	Standard deviation	1.0777E−04	1.1307E−04
Second experiment	Average	0	4.3503E−03
	Standard deviation	0	6.2116E−03
Third experiment	Average	4.4689E−05	4.6860E−05
	Standard deviation	9.0969E−07	9.7955E−07

VI. CONCLUSION

This paper discusses the construction of a novel pattern search ameliorated AOA and presents its promise comparatively in terms of IIR-based model identification. The proposed algorithm adopts the original AOA for global search and appropriately integrates the pattern search algorithm to reach a better local search. The more excellent promise of the proposed pattern search ameliorated AOA is demonstrated through well-known classical benchmark functions by comparatively presenting the statistical results using the original AOA. The welded beam design problem is also adopted as a real-world optimization problem to further demonstrate the greater capacity of the proposed algorithm by comparing the performance of the proposed approach with the reported ones in the literature. In terms of the IIR model, the identification task is considered as an optimization problem. Different systems with different ranges, such as second-order plant with a first-order IIR model, second-order plant with a second-order IIR model, and high-order plant with a high-order model, are respectively considered. Then, a comparative assessment is performed by using popular and promising meta-heuristic optimizers of AOA, particle swarm optimization, artificial bee colony algorithm, electromagnetism-like optimization algorithm, cuckoo search algorithm, and flower pollination algorithm. The statistical results obtained from the assessments confirm the excellent ability of the proposed pattern search- ameliorated AOA in terms of achieving better accuracy and robustness for the identification of the IIR model.

In addition to demonstrating the effectiveness of the proposed pattern search ameliorated AOA in the identification of IIR models, there are several potential avenues for future work in this area. One possible direction is to explore the application of the algorithm in other complex optimization problems and evaluate its performance against existing metaheuristic optimizers. Furthermore, investigating the algorithm's scalability and adaptability to handle larger and more diverse systems can contribute to its practical applicability. Additionally, incorporating additional enhancements, such as hybridization with other optimization techniques or integrating machine learning approaches, could further improve the algorithm's performance. Overall, these future research directions have the potential to expand the understanding and applicability of the pattern search ameliorated AOA in various domains.

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