

# Performance of Antenna Diversity with Turbo Trellis Coded Modulation Over Nakagami Channels

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## **ABSTRACT**

*In this paper, we will use the Space Time Block codes for antenna diversity, and the performance of Space-Time – Turbo Trellis Coded Modulation (ST-TTCM) is investigated over Nakagami channels. For high data transmission over wireless fading channels, Space-Time Block Codes provide the maximal possible diversity advantage for multiple decoding algorithms. The effects of fading channel are considered and modeled by Nakagami distribution function. Here, we investigate Space Time-Turbo Trellis Coded Modulation (ST-TTCM) for 8PSK for several Nakagami parameters  $M$ . Thus, our results will reflect the degradations both due to the effects of the fading on the amplitude of the received signal.*

**Key Words:** Turbo Trellis Coded Modulation, Space-Time Codes, Nakagami Channel

## **1. INTRODUCTION**

Turbo codes are the most efficient codes for low-power applications such as deep-space and satellite communications, as well as for interference limited applications such as third generation cellular and personal communication services. Since Turbo codes use convolutional codes as their constituent codes, a natural extension of the Turbo concept, which improves bandwidth efficiency, is its application to systems using TCM. As in [1], the main principle of turbo codes is applied to TCM by retaining the important properties and advantages of both of their structures. Just as binary turbo codes use a parallel concatenation of two binary recursive convolutional encoders, we have concatenated

two recursive TCM encoders as in [2][3], and adapted the interleaving and puncturing.

Increasing the quality or reducing the effective error rate in a multipath fading channel is extremely difficult. In Additive White Gaussian Noise (AWGN), using typical modulation and coding schemes, reducing the effective bit error rate from  $10^{-2}$  to  $10^{-3}$  may require only 1 or 2 dB higher signal-to-noise ratio. Achieving the same in a multipath fading environment may require up to 10 dB improvement in SNR [6]. The improvement in SNR may not be achieved by higher transmit power or additional bandwidth, as it is contrary to the requirements of next generation systems. It is there for crucial to effectively reduce the effect of fading at both

the remote units and the base stations, without additional power or any sacrifice in bandwidth.

Recently different transmit diversity techniques have been introduced to benefit from antenna diversity also in the downlink while putting the diversity burden on the base station. In [4] space-time trellis coding has been introduced proposing joint design of coding, modulation, transmit diversity and optimal receive diversity. Substantial benefits have been shown to be achieved by channel coding techniques appropriate to multiple transmit antenna systems. In this paper, we give a generalized scheme for concatenation of Space-Time block codes and Turbo Trellis Coded Modulation. In the decoder side, symbol-by-symbol log-map algorithm and for the inputs of the decoder, channel estimator and combiner have been taken under consideration.

This paper is organized as: in Section 2, Space Time Block Codes scheme is explained, in Section 3, the over all system, Space Time-Turbo Trellis Coded Modulation (ST-TTCM) is given and in Section 4, ST-TTCM is simulated for different Nakagami fading parameters of our system. It is shown that there causes degradation due to SNR

**2. SYSTEM MODEL**

In this Section, we investigate Space Time-Turbo Trellis Coded Modulation (ST-TTCM) over fading channel. The general block diagram of the considered scheme is given in Figure 1. Here, the main emphasis is on the performance of the ST-TTCM in fading environment modeled by Nakagami probability density function.

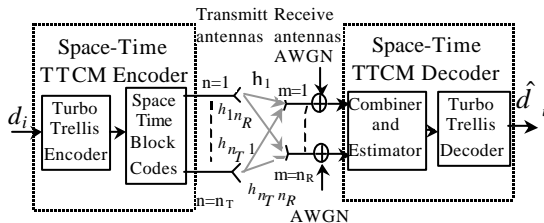


Figure 1. General Block Diagram of (ST-TTCM)

The input binary data is passed through a turbo encoder followed by a Space-Time block encoder which is shown in Figure 2. Furthermore at the transmitter side,  $n_T$  number of antennas are placed in order to achieve delay diversity and the Figure 3 shows the space-time block codes. The

channel is modeled as Nakagami pdf. The amplitude and phase distortion of the channel is carried out by  $h$  parameters shown in Figure 4. At the receiver side, the distorted multi-path signals arrive to at  $n_R$  number of receiver antennas. At the decoder of ST-TTCM scheme, turbo-trellis decoder follows combiner and estimator. Then  $\hat{d}_i$  data sequence is estimated.

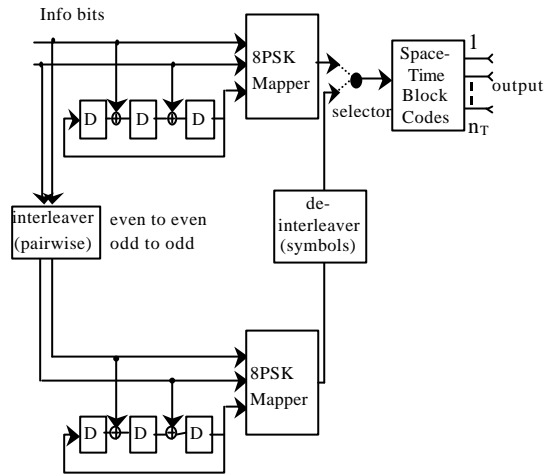


Figure 2. Encoder of ST-TTCM for 8PSK

**2.1 Space-Time Block Codes**

In wireless communication, multipath fading results severe amplitude and phase distortion. So it is crucial to combat the effect of the fading at both the remote units and the base stations, without additional power or any sacrifice in bandwidth. Since the transmitter has no information about the channel fading characteristics, it has to be fed from the receiver to the transmitter, which results complexity to both transmitter and receiver. In some cases, there may be no link in between them. There are some effective techniques, which are time and frequency diversity. Time interleaving may result in large delays. Spread spectrum approaches are ineffective if the coherent bandwidth of the channel is larger than the spreading bandwidth. So we can conclude that in fading environments, antenna diversity is a practical, effective and, hence a widely used technique for reducing the effect of multipath fading [5][6]. Space-time coding is a bandwidth and power efficient method of communication over Nakagami fading channels that realizes the benefits of multiple transmit antenna [4].

Figure 3 shows an example for a space-time block encoder. Input of a Space-Time block encoder is a block of  $M$  complex  $x_i$ ,  $i=1\dots M$ , where  $x_i$  are elements of a higher order modulation constellation, e.g. M-PSK. The space-time block encoder maps the input symbols on entries of a  $p \times n_T$  matrix  $\mathbf{G}$ , where  $n_T$  is the number of transmit antennas. The entries of the matrix  $\mathbf{G}$  are the  $M$  complex symbol  $x_i$ , their complex conjugate  $x_i^*$  and linear combinations of  $x_i$  and  $x_i^*$ . The  $p \times n_T$  matrix  $\mathbf{G}$  which defines the Space-Time Block Code is a complex generalized orthogonal as defined [5], which means that the columns of  $\mathbf{G}$  are orthogonal. An examples for  $n_T = 2$  is the complex generalized orthogonal design

$$\mathbf{G}_1 = \begin{bmatrix} g_{11} & \Lambda & g_{1n_T} \\ M & & M \\ g_{p1} & \Lambda & g_{pn_T} \end{bmatrix} = \begin{bmatrix} x_1 & x_2 \\ -x_2^* & x_1^* \end{bmatrix} \quad (1)$$

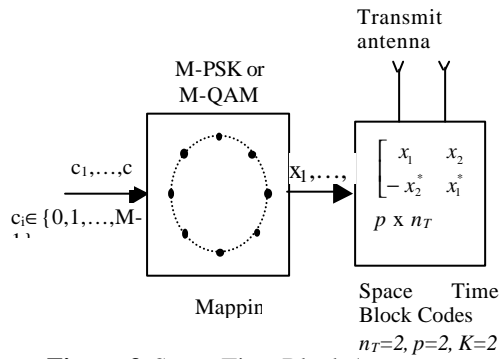


Figure 3. Space Time Block Code Encoder

In this paper, we consider a simple transmit diversity scheme which improves the signal by simple processing across two transmitter antennas and one receiver antenna as shown in Figure 4. We assume ST-TTCM modulated two signals are simultaneously sent from the two transmitter antennas,  $tx_0$ ,  $tx_1$ . In the first coding step, the signal transmitted from the first antenna,  $tx_0$  is denoted as  $s_0$  and from the second antenna  $tx_1$  as  $s_1$ . In the second coding step; antenna  $tx_0$ , transmits  $(-s_1^*)$ , while antenna  $tx_1$ , transmits  $s_0^*$  where  $*$  means complex conjugate operation. Let assume that Nakagami channel parameters is constant during these two consecutive symbols. The Nakagami

channel in between the transmitter antenna,  $tx_0$  and receiver antenna is defined as  $h_0$ . The channel between the transmitter antenna,  $tx_1$  and the receiver is defined as  $h_1$ . The multipath channels  $h_0$  and  $h_1$  are modelled as,

$$h_0(t) = h_0(t+T) = \mathbf{r}_0 e^{j\mathbf{q}_0}$$

$$h_1(t) = h_1(t+T) = \mathbf{r}_1 e^{j\mathbf{q}_1} \quad (2)$$

where  $T$  is the symbol duration.  $\mathbf{r}$  is fading amplitude and the term  $e^{j\mathbf{q}}$  is a unit vector where  $\mathbf{q}$  represents the phase noise as mentioned in [7]

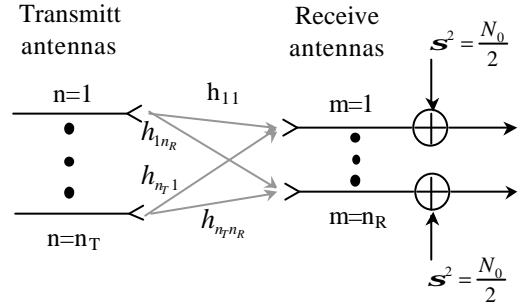


Figure 4. Space-Time Antenna Configuration

In this paper, we investigate bit error performance of ST-TTCM over Nakagami environment. Assuming that fading parameter  $\rho_i$  is independent of coding step  $i$ , Nakagami probability density function can be written as,

$$P(\mathbf{r}) = \frac{2}{\Gamma(M)} \left( \frac{M}{P_0} \right)^M \mathbf{r}^{2M-1} e^{-\frac{M\mathbf{r}^2}{P_0}} \quad (3)$$

where  $M$  is a shape parameter and defined as

$$M = \frac{P_0^2}{E((\mathbf{r}^2 - P_0)^2)} \quad \text{and} \quad P_0 = E(\mathbf{r}^2).$$

By altering  $M$ -parameter of Nakagami, many physical and practical channels can be modelled perfectly. Nakagami pdf becomes Rayleigh pdf when  $M=1$ , Rician pdf when  $M>1$  and AWGN when  $M$  is  $\infty$ .

The received signals can be expressed as [6],

$$r_o = r(t) = h_o s_o + h_1 s_1 + n_o$$

$$r_1 = r(t+T) = -h_o^* s_1 + h_1^* s_o + n_1 \quad (4)$$

where  $r_o$  and  $r_1$  are received signals at time  $t$  and  $t+T$  and  $n_o$  and  $n_1$  are Gaussian Noise with

$$\text{the noise variance is } \sigma^2 = \frac{N_o}{2E_s}.$$

The combiner in Figure 5, forms the following signals which are fed to ST-TTCM decoder.

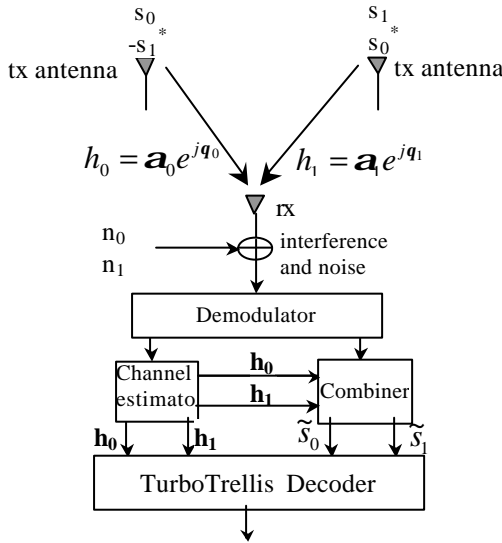


Figure 5. Receiver Side Block Diagram of ST-TTCM

$$\approx s_o = h_o^* r_o + h_1 r_1^*$$

$$\approx s_1 = h_1^* r_o - h_o r_1^* \quad (5)$$

Substituting Equations (4) and (5), we get,

$$\approx s_o = (r^2_o + r^2_1)s_o + h_o^* n_o + h_1 n_1^*$$

$$\approx s_1 = (r^2_o + r^2_1)s_1 - h_o n_1^* + h_1 n_o \quad (6)$$

The output of Equation (6) are now entries of our Turbo Trellis decoder (Figure 6).

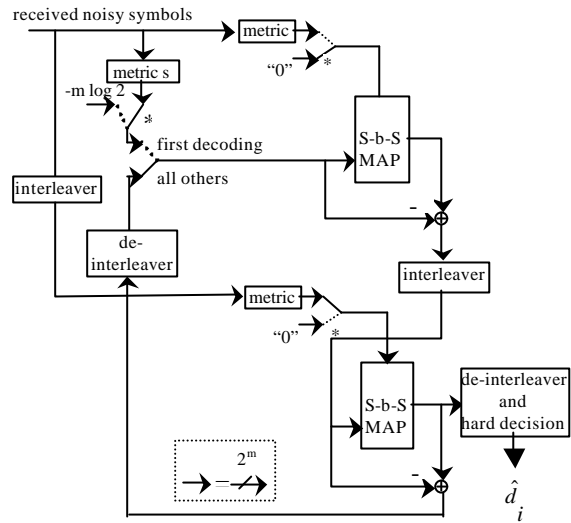


Figure 6. Decoder of ST-TTCM for 8PSK

### 3. SPACE-TIME - TURBO TRELLIS CODED MODULATION SCHEME

In this Section, we investigate the performance of Space-Time - Turbo Trellis Coded Modulation (ST-TTCM) over AWGN and Nakagami channels. The block diagram of the ST-TTCM system operating over Nakagami fading environments is shown in Figure 1. We present Space Time Block Codes for Turbo Trellis Coded signals. Space Time coding achieves higher error performance without increasing bandwidth by using various combinations of antennas diversity. As we know Turbo Trellis Coded Modulation (TTCM) is similar to binary turbo codes, but employs Trellis Coded Modulation (TCM) codes which include multi-dimensional codes. The combination of turbo codes with trellis codes leads to a straightforward encoder structure, and allows iterative decoding as binary turbo decoder. However, iterative Turbo Decoder needs to be adapted to the decoding of TCM codes. Here, we investigate ST-TTCM for 8PSK for several Nakagami parameters.

#### 3.1 ST-TTCM Encoder

Turbo codes use convolutional codes as their constituent codes, a natural extension of the Turbo concept, which improves bandwidth efficiency is its application to systems using TCM. As in [2][3], the main principle of turbo codes is applied to TCM by retaining the important properties and advantages of both of their structures. Essentially, TCM codes can be

seen as systematic feedback convolutional codes followed by one (or more for multi dimensional codes) signal mapper(s). Just as binary turbo codes use a parallel concatenation of two binary recursive convolutional encoders, we have concatenated two recursive TCM encoders as in [2], and adapted the interleaving and puncturing.

The most important characteristic of turbo codes is their simple use of recursive systematic component codes in a parallel concatenation scheme. Pseudo-random bit-wise interleaving between encoders ensures a small bit-error probability [4]. In [2], Ungerboeck codes (and multidimensional TCM codes) have been employed as building blocks in a Turbo Coding scheme in a similar way as binary codes were used.

We use the encoder which is shown in Figure 2 and introduced in [2] and [3], and we will not give any more detail for this known structure.

### 3.2 ST-TTCM Decoder

The Turbo Trellis Decoder is similar to that used to decode binary turbo codes, except there is a difference passed from one decoder to the other, and the treatment of the very first decoding step. In binary turbo coding, the decoder's output can be split into three parts. These parts are systematic component, a priori component, and extrinsic component [7]. But only the extrinsic component may be given to the next decoder; otherwise, information will be used more than once in the next decoder [1].

Here, for the Turbo Trellis Decoder the systematic information is transmitted together with parity information in the same symbol. However, we can split the decoder output into two different components, first one is a priori and the second is extrinsic and systematic together.

#### 3.2.1 Metric Calculation

Matrix calculation was used in the very first decoding stage as in [2]. We have relied on the fact that if the upper decoder sees a group of  $n$  punctured symbols, we have embedded the systematic information in the a priori input (Figure 6). Before the first decoding pass off the upper decoder, we need to set the a priori information to contain the systematic information for the  $*$  transitions, where the

symbol is transmitted partly by the information group  $d_k$ , but also the unknown parity bit  $b_k^{0,*} \in \{0,1\}$  produced by the other encoder. We set the a priori information, by applying the mixed Bayes' rule, to

$$\begin{aligned} \Pr\{d_k = i | y_k\} &= \text{const } p(y_k | d_k = i) \\ &= \text{const } \sum_{j \in \{0,1\}} p(y_k, b_k^{0,*} = j | d_k = i) \\ &= \frac{\text{const}}{2} \cdot \sum_{j \in \{0,1\}} p(y_k | d_k = i, b_k^{0,*} = j) \end{aligned} \quad (7)$$

where it is assumed that  $\Pr\{b_k^{0,*} = j | d_k = i\} = \Pr\{b_k^{0,*} = j\} = 1/2$  and  $\mathbf{y}_k = (y_k^0, \dots, y_k^{(n-1)})$  if the receiver observes  $N$  set of  $n$  noisy symbols, where  $n$  such symbols are associated with each step in the trellis. In Equation 7, it is not necessary to calculate the value of the constant since the value of  $\Pr\{d_k = i | y_k\}$  can be determined by dividing the summation  $\sum_{j \in \{0,1\}}$  by its sum over all  $i$  (normalization). If the upper decoder is not at the  $*$  transition, then we set  $\Pr\{d_k = i\}$  to  $1/2^m$  where  $m$  is the number of Turbo Trellis encoder input.

First of all the state transitions must be calculated by the given formulation below,

$$\begin{aligned} \mathbf{g}(y_k, M', M) &= p(y_k | d_k = i, S_k = M, S_{k-1} = M') \\ &\cdot q(d_k = i | S_k = M, S_{k-1} = M') \\ &\cdot \Pr\{S_k = M | S_{k-1} = M'\} \end{aligned} \quad (8)$$

$q(d_k = i | S_k = M, S_{k-1} = M')$  is either zero or one, depending on whether encoder input  $i \in \{0, 1, \dots, 2^m - 1\}$  is associated with the transition from state  $S_{k-1} = M'$  to  $S_k = M$  or not. In the last component of Equation 8 we use the a priori information

$$\Pr\{S_k = M | S_{k-1} = M'\} = \Pr\{d_k = j\} \quad (9)$$

where  $j: q(d_k = j | S_k = M, S_{k-1} = M') = 1$ . If there does not exist a  $j$  such that  $q(d_k = j | S_k = M, S_{k-1} = M') = 1$ , then  $\Pr\{S_k = M | S_{k-1} = M'\}$  is set to zero

Now let  $\bar{\mathbf{g}}_i(s_k \rightarrow s_{k+1})$  be the natural logarithm

$$\begin{aligned} &\text{of } \mathbf{g}_i(s_k \rightarrow s_{k+1}) \\ \bar{\mathbf{g}}_i(s_k \rightarrow s_{k+1}) &= \ln \mathbf{g}_i(s_k \rightarrow s_{k+1}) \end{aligned} \quad (10)$$

Now let  $\bar{\mathbf{a}}(s_k)$  be the natural logarithm

$$\begin{aligned} &\text{of } \mathbf{a}(s_k), \\ \bar{\mathbf{a}}(s_k) &= \ln \mathbf{a}(s_k) \end{aligned}$$

$$= \ln \left\{ \sum_{s_{k-1} \in A} \exp [\bar{\mathbf{a}}(s_{k-1}) + \bar{\mathbf{g}}_i(s_{k-1} \rightarrow s_k)] \right\} \quad (11)$$

where  $A$  is the set of states  $s_{k-1}$  that are connected to the state  $s_k$ .

Now let  $\bar{\mathbf{b}}(s_k)$  be the natural logarithm of  $\mathbf{b}(s_k)$ ,

$$\begin{aligned} \bar{\mathbf{b}}(s_k) &= \ln \mathbf{b}(s_k) \\ &= \ln \left\{ \sum_{s_{k+1} \in B} \exp [\bar{\mathbf{b}}(s_{k+1}) + \bar{\mathbf{g}}_i(s_k \rightarrow s_{k+1})] \right\} \quad (12) \end{aligned}$$

where  $B$  is the set of states  $s_{k+1}$  that are connected to state  $s_k$ . Therefore, the desired output of the MAP decoder is

$$P_r\{d_k = i | \mathbf{y}\} = \text{const} \cdot \sum_{M'} \sum_{M''} [\bar{\mathbf{g}}_i(y_k, M', M'') + \bar{\mathbf{a}}_{k-1}(M') + \bar{\mathbf{b}}_k(M'')] \quad (13)$$

$\forall i \in \{0, \dots, 2^m - 1\}$ . The constant can be eliminated by normalizing the sum of above formulation over all  $i$  to unity.

#### 4. SIMULATION RESULTS

In the encoder structure as shown in Figure 2, we have two information inputs, each with 1024 bit frame size. To create parity bits, both encoders are constituted of three memories. 8PSK mappers were used, and Space-Time block has been located after the puncturer. At the decoder, the signals reach to the receiver antenna from the two transmitter antennas using multipath fading environment (Figure 4 and 5). Quadrature and inphase coordinates of the received noisy signal is detected by using demodulator. Then they are processed by Space-Time decoder with channel estimator and combiner blocks. The pre-processed noisy signals are evaluated by Turbo-Trellis Decoder as shown in Figure 6. As an example, we simulate ST-TTCM system for 8PSK over AWGN and Nakagami channel. We obtain the simulation curves for different  $M$ ,  $\alpha$  and SNR values. Although Space-Time Block Codes provide up to 10dB improvement over Rayleigh channel in SNR. All the simulation results were obtained for Nakagami fading channel. The Figure 7, shows the performance of ST-TTCM signals for  $M=\infty$ . It is shown that as  $M$  improves to AWGN, bit error performance improves. Figure 8, shows the performance of TTCM signals for  $M=10$  dB. And Figure 9, shows the performance of TTCM signals for  $M=0$  dB. It is clear that for a constant iteration number, as  $M$  increases, performance improves

for the same SNR values. For  $M=0, 10, \infty$  SNR value, as iteration number increases, performance gets better. When the performance results obtained in Figure 7 through Figure 9 are compared, the degradation of error performance due to Nakagami channel parameters can easily be seen for all SNR and  $M$  values.

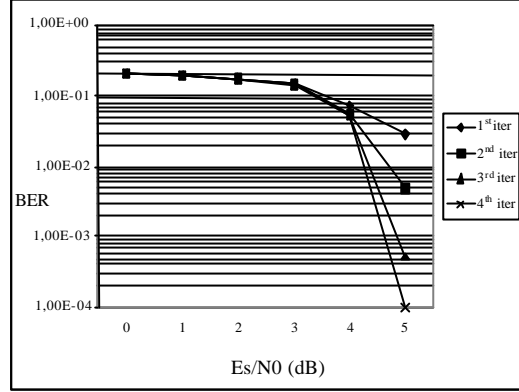


Figure 7. Bit error performance of 8PSK ST-TTCM for  $M=\infty$

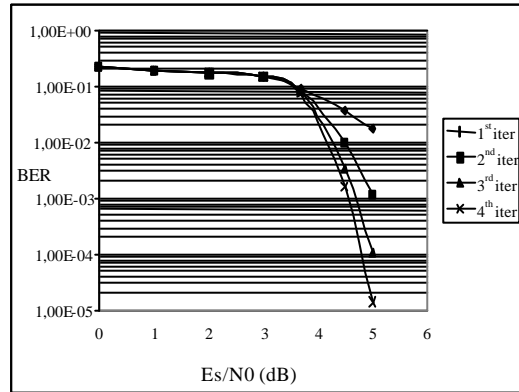


Figure 8. Bit error performance of 8PSK ST-TTCM for  $M=10$ dB

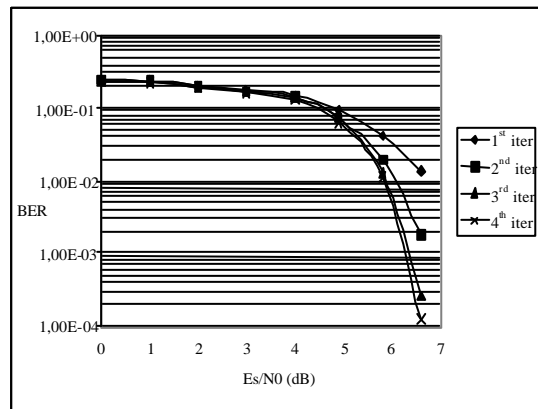


Figure 9. Bit error performance of 8PSK ST-TTCM for  $M=0$ dB,

## 5. CONCLUSION

In wireless communication, antenna diversity is very attractive because of its performance in band-limited channels. In this paper we have shown performance of space time-turbo trellis codes over Nakagami fading channel. As an example, the performance of turbo trellis coded modulated signals are simulated with different fading parameter M and iteration numbers.

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