

# DISPERSION COMPENSATION USING THE INVERSE TRANSFER FUNCTION OF FIBER OPTIC CHANNEL

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## ABSTRACT

*In this article, we examined the attenuation and chromatic dispersion induced pulse broadening during soliton pulse propagation in a single mode fiber (SMF) neglecting the nonlinearity effect. It is shown by simulation that this pulse broadening and attenuation problems can be eliminated using a proposed equalizer even the dispersion parameter of the fiber changes during pulse propagation. For this purpose, Nonlinear Schrödinger Equation (NLS) that characterizes pulse propagation in optical fibers was solved numerically in Matlab environment with the developed software using split-step Fourier method and the simulation results were shown grafically in 3-D format. Using the simulations results, it is observed that the soliton impulse is distorted by broadening and attenuation proportional to the fiber length. The proposed equalizer compansates these distorsions by narrowing pulse width and increasing the pulse level to the initial value for fiber cable lengths up to 120 km but can not be sucessful for longer fiber lengths. We are planning to apply this equalizer for OFDM signals.*

**Keywords :** Fiber optic transmission, chromatic dispersion compensation, electronic equalizer, soliton.

## 1. INTRODUCTION

In the 21st century, access to huge amounts of information has become a necessity as a result of the transition to knowledge society. In this century fiber optic (FO) communication systems are used extensively to transmit large amounts of information to long distances by light velocity, because of their higher bandwidth and low attenuation characteristics. Studies are going on to bring solutions to the

problems which limit the system performance. To increase the transmission distances, absorbing and scattering losses in FO cables are to be eliminated and the dispersion that causes broadening and distortion of optical carrier signal modulated by information during its propagation along the fiber is has to be compensated. Nowadays, fiber losses are tried to be eliminated by using erbium doped fiber optic amplifiers [1,2]. Fibers with negative dispersion coefficients are also widely used

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commercially to compensate for the dispersion [3]. Intensive research is carried out on soliton based FO communication systems to reach data rates of  $T_{bit}/s$  and long distances [4]. Solitons are a special type of pulses used in FO transmission systems which can travel very long distances without distortion. They are ultra short pulses in the wavelength of  $1.55 \mu m$  produced by InGaAsP semiconductor lasers employing active mode locking techniques [5]. In this article, an electronic channel equalizer is proposed to improve the performance of soliton based FO transmission system. A soliton is applied to the channel input in the model developed in Matlab environment. In the proposed system there exists an equalizer at the output of the single mode FO cable. How the dispersion and attenuation characteristics of this cable affect the soliton along the link has been investigated at the intervals of 1 km. The improvements achieved by the proposed equalizer have been illustrated in 3-D (three dimensional) graphs, depending on the distance and dispersion coefficient. It has been observed that by using the proposed equalizer, the problems due to dispersion and attenuation are eliminated for distances up to 120 km.

## 2- PROPAGATION OF SOLITON PULSES IN SINGLE MODE FIBERS

Soliton propagation in FO can be fully characterized by Nonlinear Schrödinger Equation (NLS) obtained from the solution of Maxwell equations [6,7].

$$\frac{\partial u}{\partial z} = -\alpha u - \beta_1 \frac{\partial u}{\partial t} - \frac{j\beta_2}{2} \frac{\partial^2 u}{\partial t^2} + \frac{\beta_3}{6} \frac{\partial^3 u}{\partial t^3} + j\gamma |u|^2 u$$

(1)

where  $u(z,t)$  stands for the amplitude of the slow-changing wave envelope and expressed as

$$u(z,t) = u(0,t) \cdot e^{(j\omega_0 t - \beta_0 t)}$$

(2)

where  $u(0,t)$  is the input pulse,  $\omega_0$  the central frequency of the carrier wave and  $\beta_0$  the propagation constant of the wave. The second,

the third and the fourth terms on the right-hand side of the equation (1) are obtained by expansion into Taylor series of the propagation constant  $\beta_0(\omega)$  of the optical wave over  $\omega_0$  carrier frequency:

$$\beta_1 = \frac{1}{v_g}$$

(3)

$$\beta_2 = -\frac{\lambda_0^2 D}{2\pi c}$$

(4)

$$\beta_3 = -\left(\frac{\lambda_0^2}{2\pi c}\right)^2 \left(\frac{2D}{\lambda_0} + \frac{dD}{d\lambda}\right)$$

(5)

where  $\alpha$  is the attenuation coefficient of the fiber,  $v_g$  the group velocity of the wave,  $D$  the dispersion parameter of the fiber (ps/nm/km),  $c$  the light velocity and  $\lambda_0$  the central wavelength of the optical pulse. The first term on the right-hand side of equation(1) corresponds to the attenuation in fiber. Of propagation constant terms,  $\beta_1$  causes pulse delay,  $\beta_2$  pulse broadening and  $\beta_3$  pulse distortion. The last term designates nonlinearity effect which varies with the light intensity in fiber.  $\gamma$  is the nonlinearity coefficient. The nonlinearity term is a function of optic light strength and can be ignored if the light intensity is low. In this case, NLS equation turns into a linear equation, of which analytical solution exists. In cases where nonlinearity term is not ignored, the solution of the equation can be obtained by numerical methods. Since only pulse broadening has been considered in this work, the effects of  $\beta_1$ ,  $\beta_3$  and  $\gamma$  have been ignored. Thus, the equation (1) can be written as

$$\frac{\partial u}{\partial z} = -\alpha u - \frac{j\beta_2}{2} \frac{\partial^2 u}{\partial t^2}$$

(6)

By taking Fourier transformation of equation(6)

$$\frac{\partial U}{\partial z} = -\alpha(\Delta\omega)U - \frac{j\beta_2}{2}(\Delta\omega)^2 U$$

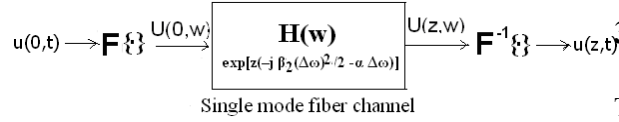
(7)

is obtained. From equation (7) the transfer function of FO cable is:

$$H(z,w)=\exp(-j z \frac{1}{2} \beta_2 (\omega - \omega_0)^2 - \alpha z (\omega - \omega_0))$$

(8)

So, the model of the linear single mode fiber channel is as shown in figure 1:



**Figure 1.**Equivalent model of a single mode fiber channel.

Here, F is the symbol for Fourier transformation,  $F^{-1}$  the symbol for inverse Fourier transformation.  $H(w)$  is the transfer function of the fiber. Output signal is defined as:

$$u(z,t)=F^{-1}\{F\{U(0,t)\}H(w)\}$$

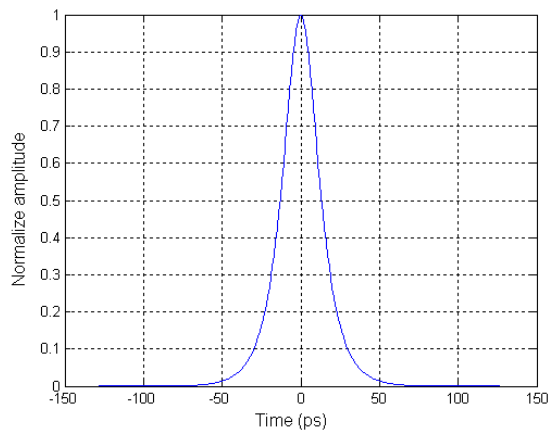
(9)

In this work, ideal solitons have been used as input pulses. Solitons are taken as

$$u(0,t)=A \operatorname{sech}(t/\tau)$$

(10)

where A is the peak amplitude of the pulse,  $\tau$  is the interval where the light intensity of soliton is reduced to the half of its peak value. Soliton applied to the fiber input used in the simulation is shown in Figure 2.

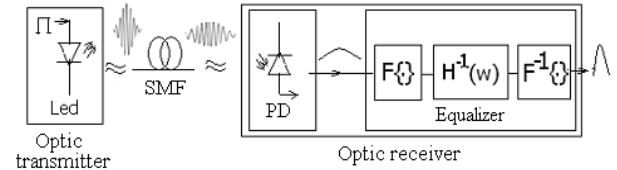


**Figure 2.** Soliton pulse applied to the fiber input.

Fourier transformation of soliton has been obtained in Matlab, then it has been multiplied by the transfer function of the fiber and finally, inverse Fourier transformation has been taken and simulated in 3-D medium.

### 3- THE MODEL OF EQUALIZER

The block diagram of optic communication system is given in Figure 3.



**Figure 3** Fiber optic transmission sysstem

The system consists of a transmitter, an FO cable, a photodiode and an equalizer. Laser and photodiode have been considered as ideal devices. In this system, the transmitter produces solitons, the FO cable carries solitons and the photodiode converts optical signals to electrical signals. The proposed equalizer carries out the Fourier transformation of the electrical signal corresponding to the envelope of the soliton coming out of the photodetector, multiplies the signal by the inverse transfer function of FO and finally obtains the inverse Fourier transformation. So, the soliton which has been cleared from dispersion and attenuation is reproduced. Any possible variation in the dispersion coefficient of FO cable is detected in the equalizer, the new dispersion coefficient is calculated and then replaced in the inverse transfer function, which has now been modified according to the new dispersion coefficient. The inverse transfer function of FO cable is defined as

$$H^{-1}(z,w)=\exp(j z \frac{1}{2} \beta_2 (\omega - \omega_0)^2 + \alpha z (\omega - \omega_0))$$

(11)

Let FO channel input signal be  $U(w,0)$  and output signal  $U(w,z)$ . In this case, the channel transfer function can be written as

$$H(z,w)=U(z,w)/U(0,w)$$

(12)

The operating principle of the equalizer is based on

$$H(w) H^{-1}(w)=1 \quad (13)$$

where  $H(w)$  is the transfer function of fiber channel (equation 8) and  $H^{-1}(w)$  is the inverse transfer function of the channel which equals the transfer function of the equalizer. The dispersion coefficient of the cable ( $D$ ) may vary depending on temperature and stress on the cable during the propagation of soliton along FO cable [ 8]. To compensate the negative impacts of this variation on the dispersion coefficient, it becomes a necessity to update the inverse transfer function by recalculating  $D$  at the equalizer input. This can be done, if one of the  $n$  different coefficient values of  $U(0,w)$  is initially known; the output coefficient value corresponding to this one i.e. the transfer function coefficient is obtained from

$$\ln(H(z,w))=\ln(\exp(-jz \ 0.5\beta_2 (w-w_0)^2 -\alpha z(w-w_0))) \quad (14)$$

Then, the new values of  $\beta_2$  and  $D$  are obtained from the equations:

$$\beta_2 = -(2(\ln(H(w,z)) + \alpha z(w-w_0)))/(jz(w-w_0)^2) \quad (15)$$

$$D = -\beta_2 2\pi c/\lambda^2 \quad (16)$$

The inverse transfer function is updated by substituting these values in the equation.

#### 4- ANALYSES AND RESULTS OF SIMULATION

In this article, the compensation of the dispersion of soliton pulse applied to the input of single mode fiber of  $z$  length by using equalizer has been simulated. Assuming that the amplitude of the pulse applied to the input is sufficiently small, nonlinearity of fiber has been ignored and only the dispersion and attenuation effect of the cable studied. The  $z$  length has varied at the intervals of 1 km.

S. KARAHAN

Table 1 shows the dispersion coefficient values ( $D1$ ) reobtained by the equalizer for different distances ranging from 103 km to 107 km, taking  $\alpha$  as 0.1 dB and  $D$ , the dispersion coefficient as 15, 17 and 20 ps/nm/km.

**Table 1.** New dispersion coefficient ( $D1$ ) obtained for different lengths ( $z$ ) and dispersion coefficients ( $D$ ) of fiber.

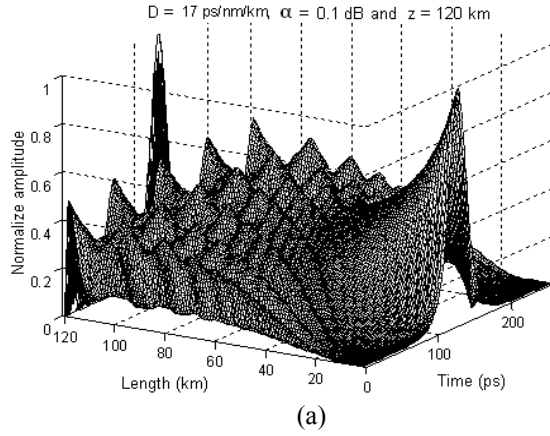
D (ps/nm/km)	$\alpha$ (dB)	z(km)	D1 (ps/nm/km)
15	0.1	119	15
15	0.1	125	15
15	0.1	130	15
15	0.1	136	15
15	0.1	137	-14.22
17	0.1	120	17
17	0.1	121	-16
20	0.1	102	20
20	0.1	103	-19
20	0.1	110	-17

The dispersion coefficient of a standard single mode FO cable is 17 ps/nm/km. To investigate the improvement of the simulated system for media having different dispersion coefficients, the analyses have also covered upper and lower values. As can be seen from Table 1:

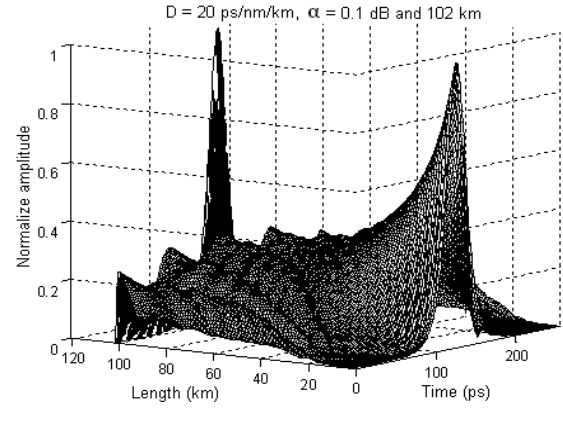
- For  $D=15$  ps/nm/km,  $D1$  equals  $D$  for distances up to 136 km. For longer distances  $D1$  takes negative values.
- For  $D=17$  ps/nm/km,  $D1$  equals  $D$  for distances up to 120 km. Error occurs in  $D1$  for longer distances.
- For  $D=20$  ps/nm/km,  $D1$  equals  $D$  for distances up to 102 km. For longer distances  $D1$  is a negative number.

Dispersion of a soliton pulse versus the distance along an FO cable of 17 ps/nm/km is illustrated in figure 4 as 3-D.

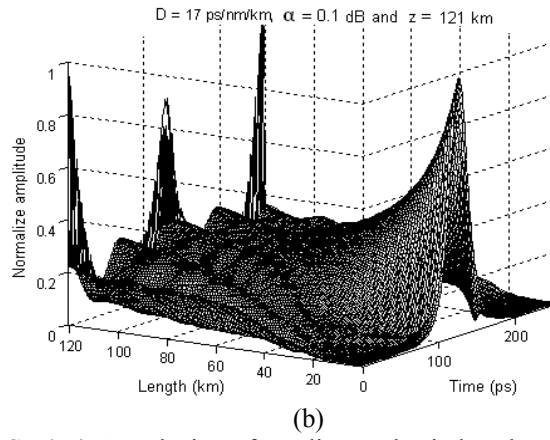
H. S. VAROL



(a)



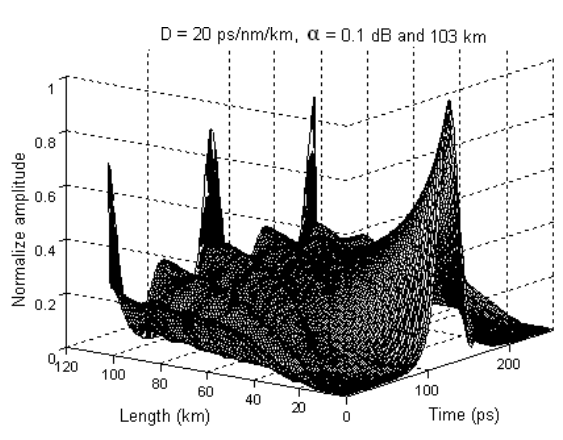
(a)



(b)

**Şekil 4.** Broadening of a soliton pulse induced by dispersion along the FO cable for  $D = 17 \text{ ps/nm/km}$ .

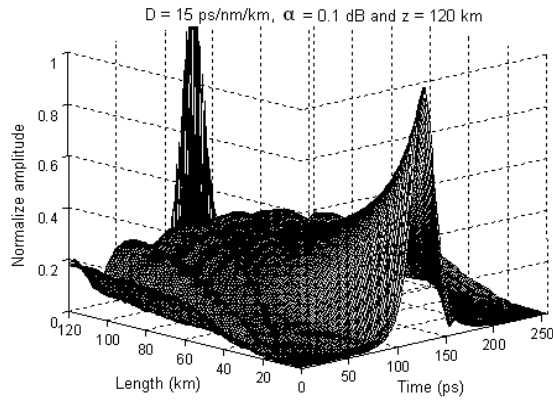
Figure 4a shows that the equalizer can compensate the dispersion forming in FO cable up to 120 km, whereas Figure 4b shows that it is incapable of compensating dispersion and attenuation effects for longer distances. Similarly, dispersion of a soliton pulse versus the distance along an FO cable of 20 ps/nm/km is given in figure 5 as 3-D.



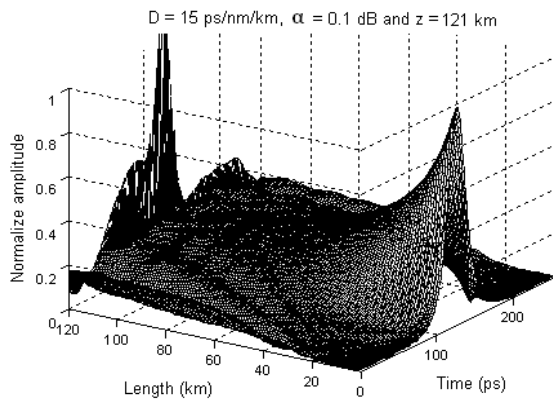
(b)

**Şekil 5.** Broadening of a soliton pulse induced by dispersion along the FO cable for  $D = 20 \text{ ps/nm/km}$ .

As seen from Figure 5a, the equalizer can compensate dispersions in FO cable up to 102 km, but for longer distances it fails as seen from figure 5b. Figure 6 gives illustration of dispersion of a soliton pulse along an FO cable of 15 ps/nm/km in 3-D.



(a)



(b)

**Şekil 6.** Broadening of a soliton pulse induced by dispersion along the FO cable for  $D = 15$  ps/nm/km.

The equalizer can compensate dispersions up to 120 km as shown in Figure 6a but fails for longer distances as indicated in Figure 6b.

## 5- CONCLUSIONS

This work has studied attenuation and broadening of a soliton pulse applied to the input of a single mode FO cable resulting from the chromatic dispersion coefficient and attenuation of FO cable. It has been demonstrated that attenuation and broadening can be compensated by the proposed electronic

equalizer using inverse transfer function of the cable. Numerical solutions and simulation results have been obtained as 3-D graphs in Matlab using the developed software program. The equalizer inverse transfer function can be updated even though dispersion coefficient of FO cable varies with temperature and similar parameters. Simulations using different dispersion coefficients result in as follows;

- For  $D=17$  ps/nm/km and  $\alpha=0.1$  dB/km, the proposed equalizer can compensate the dispersion up to 120 km.
- For  $D>17$  ps/nm/km, the compensation distance decreases, e.g. for  $D=20$  ps/nm/km it reduces to 102 km.
- For  $D<17$  ps/nm/km, the proposed equalizer can compensate the dispersion up to 120 km with success, however  $D_1$  can be calculated for longer distances.

## REFERENCES

- [1] Chapman D.A., "Erbium Doped Fiber Amplifiers: the latest revolution in fiber optical communication" Electro and Commun.Eng Journal , p 59-62, 2004.
- [2] Becker P.C., Olsson N.A., Simpson J.R., "Erbium Doped fiber Amplifiers (Optics and Photonics) Academic Pres CA, USA, 1999.
- [3] J.M.Dugan, A.J.Price, D.W.Hall, "All optical fiber based 1550 nm dispersion compansation in a 10Gb/s", Optical Fiber Communications, p14, 1992.
- [4] S.G.Mallenauer, "Experimental observation of Picosecond Pulse Broadening annd Solitons in optical fibers", Phys.Rev.Letters, Vol.45, No 13, 1980.
- [5] A.Haseagava, "Optical Solitons in Fibers" Springer-Verlag, 1989.

[6] Govind P. Agrawal A. John & sons, "Fiber Optic Communication Systems" INC. publication, NY, 2002.

[7] A.Altuncu, "Compensation of Chromatic Dispersion in Single Mode Fiber by Using DCF (in turkish)" Dumlupınar University, Journal of Science Institute, Vol:7, September, 2004.

[8] Michael J.Hamp, John Wright, Michael Hubbard, and Bob Brimacombe," Investigation into the Temperature Dependence of Chromatic Dispersion in Optical Fiber" IEEE Photonics Technology Letters 1, p 1-3, 2002.