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INSTANTANEOUS FREQUENCY ESTIMATION FOR MULTI-COMPONENT SIGNALS USING A LEAST SQUARES EVOLUTIONARY SPECTRUM

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ABSTRACT

We present a method for estimating the instantaneous frequency of the multi-component signals. This method involves the calculation of a time-frequency energy density of the signal, then obtaining an instantaneous frequency estimation from this joint density. Time-frequency energy density is calculated as a least squares optimal combination of multi-window Gabor based evolutionary spectra. The optimal weights are obtained by minimizing an error criterion that is the difference between a reference time-frequency distribution and the combination of evolutionary spectra. Instantaneous frequency of the signal is estimated from the final evolutionary spectrum as time conditional average frequency at local time-frequency regions. Examples are given to illustrate the performance of our method.

Keywords: *Instantaneous frequency, Time-frequency analysis, Evolutionary spectrum, Multi-component signals.*

1. INTRODUCTION

Instantaneous frequency (IF) of a signal, ω(t), is defined as the derivative of the phase of its corresponding analytic signal, $x(t) = A(t)e^{j\phi(t)}$ [1]. Moreover, from a joint time-frequency (TF) perspective, the IF of a signal is defined as the average of frequencies at a given time (or time conditional mean frequency) [2]:

$$<\omega>_t = \omega(t) = \int \omega \frac{S(t,\omega)}{S(t)} d\omega \qquad (1)$$

where

$$S(t) = \int S(t,\omega) d\omega$$

is the density in time $(|x(t)|^2)$ or time marginal of the TF density $S(t,\omega)$. Estimating the IF of a signal is an important issue in many signal processing applications such as communications, radar, bioengineering, etc. [3,4]. For instance, in spread spectrum communication systems, jammers can be eliminated by estimating their IF and removing them by a time-varying filter [5]. In our approach the IF is estimated using a least squares multi-window evolutionary spectrum as the TF energy density for the signal.

TF signal analysis is a helpful tool for analyzing the time-varying frequency content of a non-

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stationary signal [2]. The Wigner-Ville Spectrum (WVS) is defined as a time-dependent spectrum for non-stationary stochastic process $x(t)$ and given by [6]:

$$P(t,\omega) = E\{W(t,\omega)\}$$
$$= E\left\{ \int_{-\infty}^{\infty} \left[x\left(t-\frac{\tau}{2}\right) x^*\left(t+\frac{\tau}{2}\right) \right] e^{-j\omega\tau} d\tau \right\}$$

where $W(t,\omega)$ denotes the Wigner Distribution (WD) and the above is the statistical average of the WDs of the realizations of the process. When we have several observations of the non-stationary process $x(t)$, we can use an ensemble average of the individual WDs of these observations to estimate the WVS. However, this is not the case in general; we are only given a single realization of the process. In that case, Time-Frequency Distributions (TFDs) with a smoothing kernel function is used to estimate the WVS [2]. A good amount of research has been done to design kernels with desired properties yielding unbiased and low variance WVS estimates [6, 8].

A new estimate of the WVS is proposed as the optimal average of multiple-window spectrograms of the process in [9, 10]. In this work we use a WVS estimate that is an optimal combination of evolutionary spectra obtained by a multi-window Gabor expansion [7]. The optimal combination coefficients are obtained by minimizing the squared error between a reference TFD (which is taken to be the Wigner-Ville Distribution of the signal) and the multi-window spectral estimate.

2. EVOLUTIONARY SPECTRAL ANALYSIS

Given a non-stationary signal, $x(n), 0 \le n \le N-1$, a discrete Wold-Cramer representation [12] for it is given by

$$x(n) = \sum_{k=0}^{K-1} A(n,\omega_k) e^{j\omega_k n}, \qquad (2)$$

where $\omega_k = 2\pi k / K$, K is the number of frequency samples, and $A(n,\omega_k)$ is an evolutionary kernel. The evolutionary spectrum is obtained from this kernel as $S(n,\omega_k) = \frac{1}{K} |A(n,\omega_k)|^2$. In [7] it has been shown that the kernel can be obtained from the coefficients of a Gabor expansion. The multi-window Gabor expansion is given by [7]

$$x(n) = \sum_{m=0}^{M-1} \sum_{k=0}^{K-1} a_{i,m,k} h_i(n-mL) e^{j\omega_k n} \qquad (3)$$
$$= \sum_{k=0}^{K-1} A_i(n,\omega_k) e^{j\omega_k n} \qquad (4)$$

where $\{a_{i,m,k}\}$ are the Gabor coefficients, $\{h_{i,m,k}\}$ are the Gabor basis functions that are obtained from a Gauss window function by scaling, translating and modulating:

$$h_{i,m,k}(n) = h_i(n-mL) e^{j\omega_k n} \qquad (5)$$

and the synthesis window $h_i(n)$ is obtained by scaling a unit-energy mother window $g(n)$ as $h_i(n) = 2^{i/2} g(2^i n), i = 0,1,\ldots, I-1.$

The multi-window Gabor coefficients are evaluated by

$$a_{i,m,k} = \sum_{n=0}^{N-1} x(n) \gamma_i^*(n-mL) e^{-jw_k n}, \qquad (6)$$

where the analysis window $\gamma_i(n)$ is solved from the bi-orthogonality condition between $h_i(n)$ and $\gamma_i(n)$ [7]. Hence by comparing the representations of the signal in (3) and (4) we obtain the evolutionary kernel as

$$A_i(n,\omega_k) = \sum_{m=0}^{M-1} a_{i,m,k} h_i(n-mL) \qquad (7)$$

Replacing for the coefficients $\{a_{i,m,k}\}$, we obtain also that

$$A_i(n,\omega_k) = \sum_{l=0}^{N-1} x(l) w_i(n,l) e^{-j\omega_k l}, \qquad (8)$$

where we defined the time-varying window for scale i as

$$w_i(n,l) = \sum_{m=0}^{M-1} \gamma_i^*(l-mL)h_i(n-mL).$$

Then the evolutionary spectrum of $x(n)$ calculated using the window $h_i(n)$ is obtained by

$$S_i(n,\omega_k) = \frac{1}{K}\left|A_i(n,\omega_k)\right|^2,$$

where the factor $1/K$ is used for proper energy normalization. We should mention that normalizing the $w_i(n,l)$ to unit energy, the total energy of the signal is preserved thus justifying the use of $S_i(n,\omega_k)$ as a TF energy density for $x(n)$. Furthermore, $S_i(n,\omega_k)$ is always non-negative and approximates the marginal conditions [2]; hence, in contrast to many TFDs, interpretable as TF energy density function [7].

3. IF ESTIMATION

Estimation of instantaneous frequency is a complex and not well understood task [1,13]. Conventionally, the IF of a mono-component signal is obtained from its time–frequency distribution function as the average of frequencies present in the signal at a given time [2]. For a multi-component signal such a computation of the IF does not have the same significance [1]. Furthermore, the usual definition of the IF being the derivative of the phase of the corresponding analytical signal fails (or do not approach our intuition) in the case of multi-component signals. The evolutionary spectrum can be used to obtain a general definition of IF by considering the signal *x(n)* as a sum of analytic signals with time-dependent magnitudes and phases, that is

$$x(n) = \sum_k |X(n,\omega_k)|\, e^{j\Psi(n,\omega_k)},$$

where $\Psi(n,\omega_k)$ = Arg $[X(n,\omega_k)]$ + ω_k n. Computing $\Psi(n,\omega_k)$ only where | $X(n,\omega_k)$ | is significant, a general instantaneous frequency function is defined as:

$$\mathrm{IF}(n,\omega_k) = \Psi(n,\omega_k) - \Psi(n-1,\omega_k). \qquad (9)$$

This can be accomplished by determining the instantaneous phase at the peaks of the spectra. On the other hand, as we will see, decomposing the signal into its components $|X(n,\omega_k)|\, e^{j\Psi(n,\omega_k)}$, these are analytic functions that will also provide the instantaneous frequency.

Consider a multi-component signal. The estimation of the signal IF is complicated by the multi-component nature of the signal. We need thus to separate the different components. The estimation is especially difficult at places where there is overlap of the spectra of the signal components.

4. IF ESTIMATION BY LEAST SQUARES EVOLUTIONARY SPECTRUM

Given a realization of a discrete-time, nonstationary process corrupted by additive noise $x(n) = s(n) + \eta(n)$ where $s(n)$ and $\eta(n)$ denotes the signal and noise processes respectively. We intend to obtain a high resolution evolutionary spectral estimate with good performance in low signal to noise ratio (SNR) conditions such that the IF of the signal $s(n)$ can be estimated. We calculate a weighted average combination of evolutionary spectra $S_i(n,\omega_k)$ that is closest to a reference TFD in a least squares sense. Given the signal $x(n)$, we calculate evolutionary spectra $S_i(n,\omega_k)$ for $i = 0,1,\ldots,I-1$ as

$$S_i(n,\omega_k) = \frac{1}{K}\left|\sum_{l=0}^{N-1} x(l)W_i(n,l)e^{-j\omega_k l}\right|^2. \qquad (10)$$

Gauss windows are used as $h_i(n)$, for their optimal concentration in the TF plane [7]. Then we estimate the WVS of the process $x(n)$ as a weighted average of the evolutionary spectra

$$\hat{P}(n,\omega_k) = \sum_{i=0}^{I-1} c_i S_i(n,\omega_k) \qquad (11)$$

where the weights $\{c_i\}$ are obtained by minimizing the error function

$$\varepsilon_i = \sum_{n=0}^{N-1}\sum_{k=0}^{K-1}\left|P_R(n,\omega_k) - \sum_{i=0}^{I-1} c_i S_i(n,\omega_k)\right|^2 \quad (12)$$

and $P_R(n,\omega_k)$ is a reference TFD which is taken here as Wigner-Ville Distribution of the signal for its optimal TF resolution.
By using a matrix notation, the minimization problem in (12) can be rewritten as

$$\min_{c_i} \left\|P_R - Sc\right\|^2 \quad (13)$$

The solution of this least squares minimization problem is

$$c^o = (S^T S)^{-1} S^T P_R$$

where the superscript $'o'$ stands for optimum. Then a WVS estimate is obtained as optimal weighted average using $\{c_i^o\}$ in equation (11). Finally, we mask or threshold our estimate $\hat{P}(n,\omega_k)$ to eliminate any possible negative values caused by any negative c_i^o coefficient, and result in a non-negative time-varying spectrum, i.e.,

$$\hat{P}(n,\omega_k)^+ = \begin{cases} \hat{P}(n,\omega_k), & \hat{P}(n,\omega_k) \ge 0; \\ 0, & \hat{P}(n,\omega_k) < 0. \end{cases} \quad (14)$$

where $\hat{P}(n,\omega_k)^+$ denotes the positive-only part of the spectrum. Then the Local IF, $IF(n,\omega_k)$, of the multi-component signals can be calculated from this TF density according to equation (1) as time conditional mean frequency in PxQ dimensional small TF regions of this TF density:

$$IF(n,\omega_k) = \sum_k \omega_k \frac{\hat{P}(n,\omega_k)^+}{S(n)}, \quad (15)$$

$$(p-1)P + 1 \le n \le pP, \quad (q-1)Q + 1 \le k \le qQ$$

where $p = 1,\ldots, N/P$, $q = 1,\ldots, K/Q$.

In our simulations we obtain both TF density and Local IF estimate of several nonstationary signals.

5. EXAMPLES

To illustrate the performance of our proposed method, we consider two chirp signals. In Fig.1, we show the least squares evolutionary spectral estimate $\hat{P}(n,\omega_k)^+$ which is obtained by using $I = 4$ windows. Fig. 2 shows the global IF estimate obtained from this evolutionary spectrum. Notice that the two components of the signal cannot be discriminated in the global IF. In Fig.3 we show the local IF estimate of this two-component signal.

In the second example, we consider the combination of a sinusoidal FM signal and a sinusoidal signal. Fig. 4 shows the initial least squares evolutionary spectral estimate of this two component signal. In Fig. 5 we show the global IF estimate obtained from the least squares evolutionary spectrum. In Fig. 6, we show the local IF estimate of this signal. It can be seen from the figures that the local IF estimate can resolve the components of the signal.

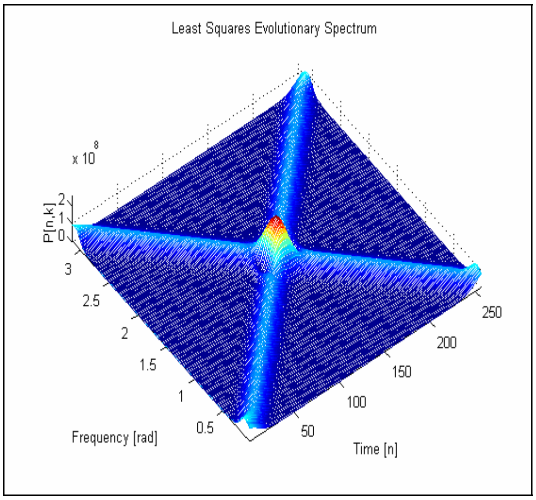

Fig. 1. *Least squares spectral estimate.*

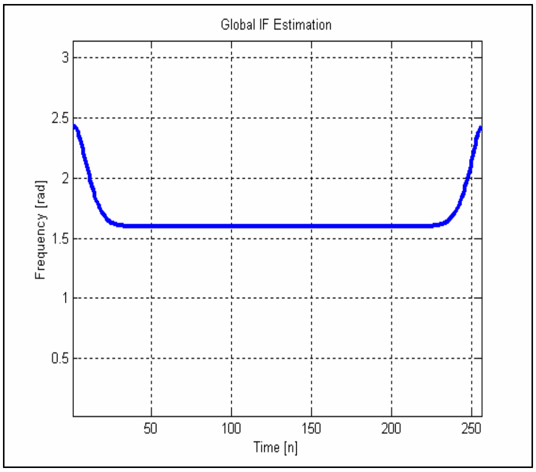

Fig. 2. *Global IF estimation*

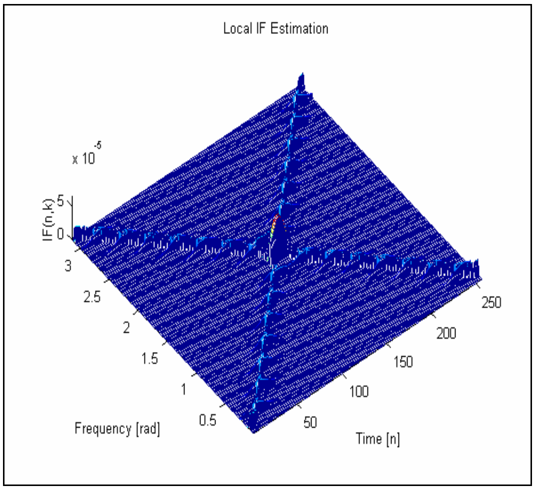

Fig. 3. *Local IF estimation for multi-component signal.*

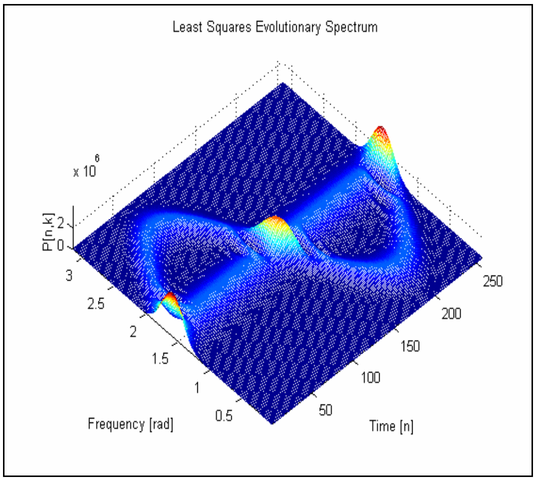

Fig. 4. *Least squares evolutionary spectral estimate.*

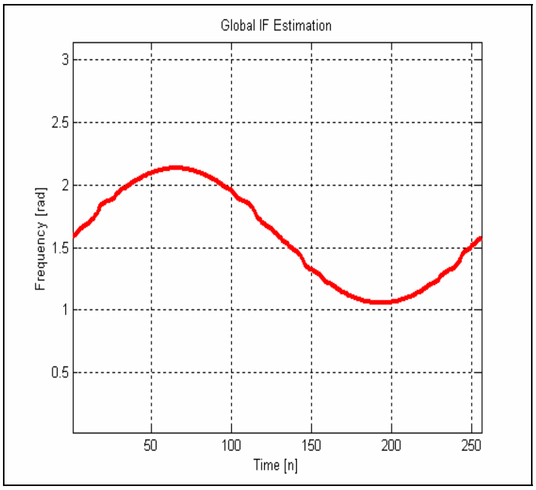

Fig. 5. *Global IF estimation*

As a final example, we consider the combination of a sinusoidal FM and two chirp signals. In Fig.7 we show the least squares evolutionary spectral estimate of this three-component signal. Fig.8 shows the global IF estimate obtained from the least squares evolutionary spectrum. Fig. 9 shows the local IF estimate. As shown, the proposed evolutionary spectrum based local IF estimation method gives better estimation results than the classic global time-conditional IF estimation method.

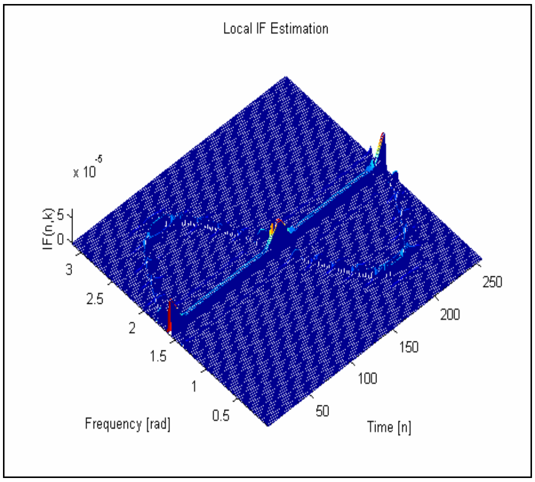

Fig. 6. *Local IF estimation for multi-component signal.*

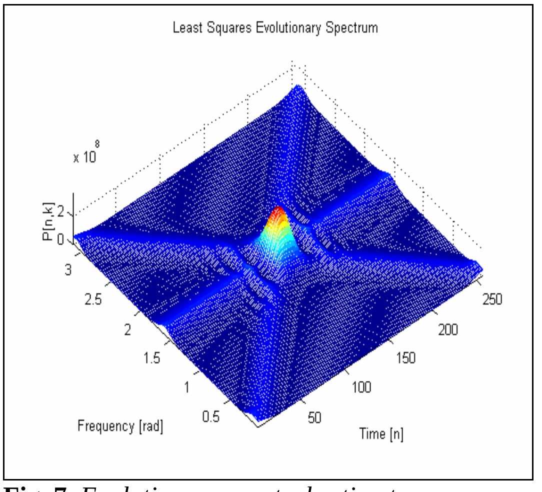

Fig. 7. *Evolutionary spectral estimate*

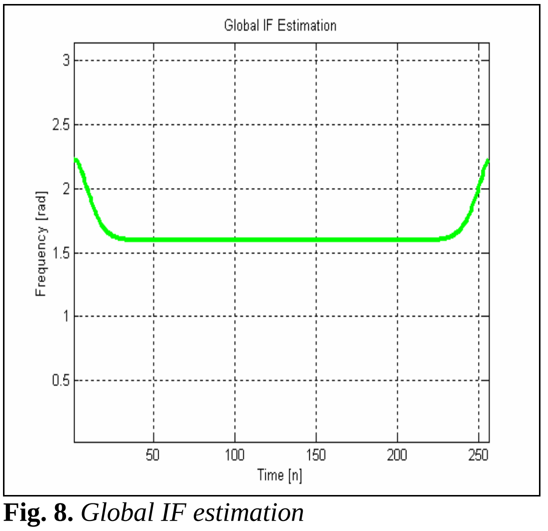

Fig. 8. *Global IF estimation*

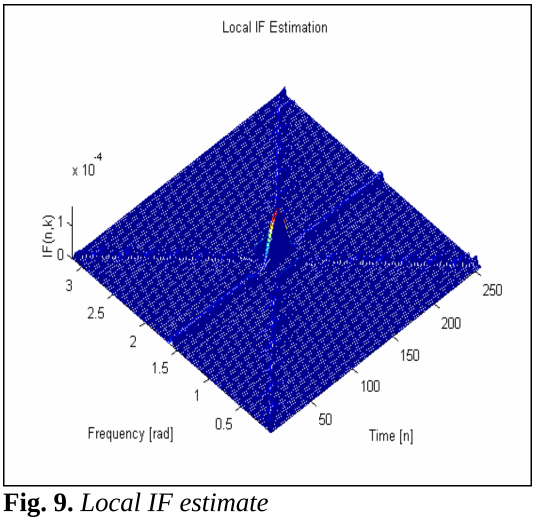

Fig. 9. *Local IF estimate*

6. CONCLUSIONS

In this work, we present a new method for obtaining the Instantaneous Frequency of non-stationary multi-component signals. Our method uses the optimal combination in the least squares sense, of evolutionary spectra that are calculated by multi-window Gabor expansion. The optimal weights are obtained by minimizing the squared error between the combination of evolutionary spectra and a reference TFD. Examples show that our method combines the advantages of multiple-window evolutionary spectral analysis and high resolution TFDs, i.e., it provides non-negative and high resolution time-varying spectral estimates. Thus, the local IF estimation can give the IF of each component of the signal. But with the classic global IF estimation method, only one IF function can be obtained for all components of the signal.

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