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Observer Design With Better Delay Margin for Linear Time-Delay Systems

Aminul HAQ[1] and Ibrahim Beklan KUCUKDEMIRAL[2]

[1] Department of Control and Automation Engineering, Yildiz Technical University, Istanbul, Turkey
[2] Department of Control and Automation Engineering, Yildiz Technical University, Istanbul, Turkey
ocean_blue04@yahoo.com, beklan@yildiz.edu.tr

Abstract: In this paper a H_∞ type observer is proposed for linear time delay systems with delay in states. The stability of the observer is proved by Lyapunov approach. The novelty of the study is to include the state derivatives in the design.As a result, better delay margin and relaibility is obtained.Two numerical examples have been illustreted to Show the validity and effectiveness of this prescribed approach and a comparison table shows the achievement of better delay margin in comparison with corresponding Luenberger type observer.
Keywords: *H_∞ observer, Lyapunov, LMI, Time-delay.*

1. Introduction

Time-delay system (TDS) is a system having delays in its states, inputs or outputs and occurs in many natural and engineering events. Time-delay is commonly encountered in chemical processes, biological systems, hydraulic systems etc and usually a very common source of instability. TDS actually belongs to the class of functional differential equation (FDE), which has infinite dimensions. making it more complex. Consideration of delay terms in system analysis[14] and designs is necessary for engineers to make models to behave like more to real process.

H_∞ observer design is one of the fruitful research area and has an inmate connection with fundamental system concepts. Last few decades different methods such as Riccati Equation approach [2,3,4], Lyapunov approach [1,6] are applied for observer design. Observer itself has different classification such as delay independent [5], delay dependent [6], delay free [8,9], positive state bounding [13]. Due to advances in computational capability, Linear Matrix Inequality (LMI) [15] is greatly used to analysis the stability of TDS. It is well known that H_∞ filtering problem is dual to the H_∞ control one for linear systems without uncertainty. H_∞ Controller (observer) design procedure has been proposed and developed in [7, 10, 11, 12], which could be adopted for observer design too because of duality. The main motivation for the study stems from the fact that if PD (Proportional differential) controller is better than only ”proportional” controller then, why not thinking of Proportional-differential type of observer design and developing it in LMI structure. The proposed state estimation scheme is based on several concepts. This observer is the result of integration of following 3 ideas to be named Lyapunov-Krasovskii Theory, Luenberger Observer, Linear Matrix Inequality.

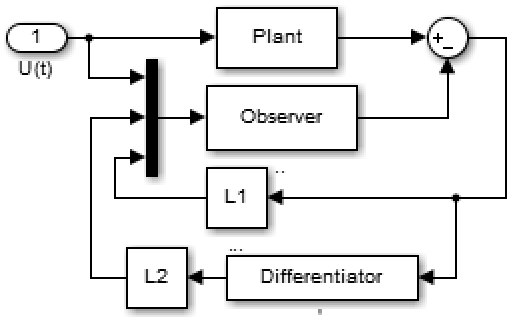

Fig 1. Block diagram of proposed observer

To design an observer for TDSS we use simple Luenberger approach, but we introduced here two feedback line instead of one. The first feedback line contains a proportional gain matrix (L_1) and second feedback line has a gain matrix (L_2, given) followed by a differentiator block. So here we are considering not only the difference between real states and estimator states or error signals but also the rate of change of error signals. Taking into consideration both error and rate of change of error data would make the observer more reliable than simple Luenberger type one.

2. Problem Formulation

Consider the following linear time-delay system,

$$\dot{x}(t)=Ax(t)+A_d x(t-h)+Bu(t)+Nw(t)$$
$$y=Cx(t)$$

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$x(t+\theta)=\varphi(\theta) \quad \forall\, \theta \epsilon[-h,0]$ (1)

$x(t)\epsilon \mathbf{R}^n$: The State vector

$w(t)\epsilon \mathbf{R}^q$: The exogenous disturbance input which belongs to $L_2[0,\infty)$.

$y(t)\epsilon \mathbf{R}^p$: The output vector.

A, $A_d\, \epsilon \mathbf{R}^{n\times n}$, $B\, \epsilon \mathbf{R}^{n\times q}$, $N\, \epsilon \mathbf{R}^{n\times k}$, $C\, \epsilon \mathbf{R}^{p\times n}$.

The above matrices are constant and known system matrices.

$h>0$:a positive scalar denoting the time delay.

$\varphi(.)$: a continuously differentiable function on $[-h,0]$ representing the initial condition.

3. Main Result

Let us formulate an observer dynamics as follows,

$$\dot{\hat{x}}(t)=F\hat{x}(t)+G\hat{x}(t-h)+Hu(t)+Mw(t)+L_1(y(t)-\hat{y}(t))+L_2(\dot{y}(t)-\dot{\hat{y}}(t))$$
$$\hat{y}(t)=C\hat{x}(t) \qquad (2)$$

$\hat{x}(t)\epsilon \mathbf{R}^n$:The estimator state vector

$L_1, L_2 \epsilon \mathbf{R}^{n\times p}$:The constant observer gain matrix to be selected appropriately.

$\hat{y}(t)\epsilon \mathbf{R}^p$:The estimated output vector.

F, $G\, \epsilon \mathbf{R}^{n\times n}$, $H\, \epsilon \mathbf{R}^{n\times q}$, $M\, \epsilon \mathbf{R}^{n\times k}$, $C\, \epsilon \mathbf{R}^{p\times n}$.

Theorem: Observer in form of (2) can be constructed if there exists matrices $P=P^T>0$, $R_1=R_1^T>0$, $R_2=R_2^T>0$ and X for a given noise attenuation level γ, satisfying the following LMI,

$$\begin{bmatrix} \Omega_2 & hZ^{-T}PA_dZ & hZ^{-T}PA_dZ & hA_d{}^T & h(A^TP-C^TX^T) & 0 \\ hZ^TA_d{}^TPZ^{-1} & -hR_1 & 0 & 0 & 0 & 0 \\ hZ^TA_d{}^TPZ^{-1} & 0 & -hR_2 & 0 & 0 & 0 \\ hA_d & 0 & 0 & h(R_2-2I) & 0 & 0 \\ h(PA-XC) & 0 & 0 & 0 & h(R_1-2P) & 0 \\ 0 & 0 & 0 & 0 & 0 & -\gamma^2 I \end{bmatrix} < 0 \qquad (3)$$

where $\Omega_2=(A^TPZ^{-1} - C^TX^TZ^{-1}+A_d^TPZ^{-1}+ Z^{-T}PA- Z^{-T}XC+ Z^{-T}PA_d +C^TC$

3.1 Proof

Subtracting equation (2) from equation (1) we get,

$\dot{x}(t) - \dot{\hat{x}}(t) = Ax(t)+A_dx(t-h)+Bu(t)+Nw(t) - F\hat{x}(t)-G\hat{x}(t-h) -Hu(t)-Mw(t)-L_1(y(t)-\hat{y}(t))-L_2(\dot{y}(t)-\dot{\hat{y}}(t))$

$\dot{e}(t) =Ax(t)+A_dx(t-h)+Bu(t)+Nw(t) - F\hat{x}(t)-G\hat{x}(t-h) -Hu(t)-Mw(t)-L_1(y(t)-\hat{y}(t))-L_2(\dot{y}(t)-\dot{\hat{y}}(t))+Fx(t) +Gx(t-h)-Fx(t)-Gx(t-h)$

$\dot{e}(t) =(A-F)x(t)+(A_d-G)x(t-h)+(B-H)u(t)+(N-M)w(t) +F(x(t)- \hat{x}(t))+G(x(t-h)- \hat{x}(t-h))-L_1(Cx(t)-C\hat{x}(t)) -L_2(C\dot{x}(t)-C\dot{\hat{x}}(t))$

$\dot{e}(t) =(A-F)x(t)+(A_d-G)x(t-h)+(B-H)u(t)+(N-M)w(t) +Fe(t)+Ge(t-h)-L_1C(x(t)- \hat{x}(t)) -L_2C(\dot{x}(t)-\dot{\hat{x}}(t))$

$\dot{e}(t)+L_2C\dot{e}(t) = (A -F)x(t)+(A_d-G)x(t-h)+(B-H)u(t) +(N-M)w(t)+Fe(t)+Ge(t-h)-L_1Ce(t)$

$(I+ L_2C)\dot{e}(t) = (A -F)x(t)+(A_d-G)x(t-h)+(B-H)u(t) +(N-M)w(t)+(F-L_1C)e(t)+Ge(t-h)$

$\dot{e}(t) =(I+ L_2C)^{-1}[(A -F)x(t)+(A_d-G)x(t-h)+(B-H)u(t) +(N-M)w(t)+(F-L_1C)e(t)+Ge(t-h)]$

$\dot{e}(t) = Z[(A -F)x(t)+(A_d-G)x(t-h)+(B-H)u(t) +(N-M)w(t)+(F-L_1C)e(t)+Ge(t-h)]$

where , $Z=(I+ L_2C)^{-1}$

Here, we will choose L_2 arbitrarily and calculate the gain L_1 accordingly.Obviously,

$e(t)\rightarrow 0$ as $t\rightarrow\infty$ if the following conditions are satisfied:

(1) The system is stable and observable.

(2) $(I+ L_2C)$ is invertible.

(3) $A=F$, $A_d=G$, $B=H$, $N=M$,
Then the error dynamics reduces to,

$$\dot{e}(t) =(I+ L_2C)^{-1}[(F-L_1C)e(t)+Ge(t-h)]$$
$$\dot{e}(t) = Z[(F-L_1C)e(t)+Ge(t-h)] \qquad (4)$$

We will utilize following the Leibniz rule

Lemma 1: $A(t-h) = A(t)-\int_{t-h}^{t}\dot{A}(\alpha)d\alpha$

We will also use the following lemma in our proof

Lemma 2: $-2U^TV\leq U^TRU+V^TR^{-1}V$

Then we have the error dynamics as follows,

$\dot{e}(t) = Z[(F-L_1C)e(t)+Ge(t-h)]$

Using Leibniz rule given in Lemma 1, we can write,

$$e(t-h) = e(t)-\int_{t-h}^{t}\dot{e}(\alpha)d\alpha$$
$$= e(t)-\int_{t-h}^{t}Z[(F-L_1C)e(\alpha)+Ge(\alpha-h)]\,d\alpha$$

$$\dot{e}(t) =Z(F-L_1C)e(t) +ZG\{e(t)-\int_{t-h}^{t}Z[(F-L_1C)e(\alpha)+Ge(\alpha-h)]\,d\alpha\}$$

The error dynamics (4) is now transformed into the following equation.

$$\dot{e}(t) = Z[(F-L_1C)+G]e(t) - ZGZ\int_{-h}^{0}[(F-L_1C)e(t+\alpha)+Ge(t+\alpha-h)]\,d\alpha \quad (5)$$

$\dot{e}(t)\to 0$ as $t\to\infty$ means error in (5) tends to '0' as time evolves.

Delay-Dependent Approach: Consider the following Lyapunov–Krasovskii functional

$$V(e,t)= e(t)^T Z^{-T}PZ^{-1} e(t) + \int_{-h}^{0}\int_{t+\theta}^{t} e(\theta)^T(F-L_1C)^T R_1 (F-L_1C)e(\theta)\,d\theta.\,ds + \int_{-h}^{0}\int_{t-h+\theta}^{t-h} e(\theta)^T G^T R_2 G\, e(\theta)\,d\theta.\,ds$$

$$\begin{aligned}\dot{V}(e,t) &= \dot{e}(t)^T Z^{-T}PZ^{-1}e(t)+ e(t)^TZ^{-T}PZ^{-1}\dot{e}(t)\\ &+h\, e(t)^T (F-L_1C)^T R_1 (F-L_1C)\, e(t)\\ &-\int_{t-h}^{t} e(\theta)^T(F-L_1C)^T R_1 (F-L_1C)e(\theta)\,d\theta\\ &+ h\, e(t)^T G^T R_2 G e(t)\\ &-\int_{t-h-h}^{t-h} e(\theta)^T G^T R_2 G e(\theta)\,d\theta\end{aligned}$$

$$\begin{aligned}&= e(t)^T[(F-L_1C)+G]^T Z^T Z^{-T}PZ^{-1}e(t)\\ &+ e(t)^T Z^{-T}PZ^{-1}Z[(F-L_1C)+G]\, e(t)\\ &-2e(t)^TZ^{-T}PZ^{-1}ZGZ\\ &\quad *\int_{-h}^{0}[(F-L_1C)e(t+\theta)+Ge(t+\theta-h)]d\theta\\ &+ h\, e(t)^T(F-L_1C)^TR_1(F-L_1C)e(t)\\ &-\int_{t-h}^{t} e(\theta)^T(F-L_1C)^T R_1(F-L_1C)e(\theta)\,d\theta\\ &+ h\, e(t)^TG^TR_2Ge(t)-\int_{t-h-h}^{t-h} e(\theta)^T G^T R_2 G e(\theta)\,d\theta\end{aligned}$$

$$\begin{aligned}&\le e(t)^T[(F-L_1C)+G]^TPZ^{-1}e(t)\\ &+ e(t)^T Z^{-T}P[(F-L_1C)+G]e(t)\\ &+\int_{-h}^{0} e(t)^T Z^{-T}PGZR_1^{-1}Z^TG^TPZ^{-1}e(t)\,d\theta\\ &+\int_{-h}^{0} e(t+\theta)^T(F-L_1C)^TR_1(F-L_1C)e(t+\theta)\,d\theta\\ &+\int_{-h}^{0} e(t)^T Z^{-T}PGZR_2^{-1}Z^TG^TPZ^{-1}e(t)\,d\theta\\ &+\int_{-h}^{0} e(t+\theta-h)^TG^TR_2Ge(t+\theta-h)\,d\theta\\ &+ h\, e(t)^T(F-L_1C)^TR_1(F-L_1C)e(t) + h\, e(t)^TG^TR_2Ge(t)\\ &-\int_{-h}^{0} e(t+\theta)(F-L_1C)^TR_1(F-L_1C)e(t+\theta)\,d\theta\\ &-\int_{-h}^{0} e(t+\theta-h)^TG^TR_2Ge(t+\theta-h)\,d\theta\end{aligned}$$

$$\begin{aligned}&\le e(t)^T[F^TPZ^{-1}-C^TL_1^TPZ^{-1}+G^TPZ^{-1}+Z^{-T}PF- Z^{-T}PL_1C\\ &\quad +Z^{-T}PG]e(t)+he(t)^TZ^{-T}PGZR_1^{-1}Z^TG^TPZ^{-1}e(t)\\ &\quad +h\, e(t)^TZ^{-T}PGZR_2^{-1}Z^TG^TPZ^{-1}e(t)\\ &\quad +he(t)^T(F-L_1C)^TR_1(F-L_1C)e(t)+ h\, e(t)^TG^TR_2Ge(t)\end{aligned}$$

Now applying Schur complement we get,

$$\le e(t)^T\begin{bmatrix}\Omega & hZ^{-T}PGZ & hZ^{-T}PGZ & hG^T & h(F-L_1C)^T\\ hZ^TG^TPZ^{-1} & -hR_1 & 0 & 0 & 0\\ hZ^TG^TPZ^{-1} & 0 & -hR_2 & 0 & 0\\ hG & 0 & 0 & -hR_2^{-1} & 0\\ h(F-L_1C) & 0 & 0 & 0 & -hR_1^{-1}\end{bmatrix} e(t)$$

Here,
$\Omega= (F^TPZ^{-1} -C^TL_1^TPZ^{-1}+G^TPZ^{-1}+ Z^{-T}PF- Z^TPL_1C+Z^{-T}PG)$

If above matrix is less than 0,then $\dot{V}(e,t)$ is negative so $e(t)\to 0$ as $t\to\infty$.

$$\begin{bmatrix}\Omega & hZ^{-T}PGZ & hZ^{-T}PGZ & hG^T & h(F-L_1C)^T\\ hZ^TG^TPZ^{-1} & -hR_1 & 0 & 0 & 0\\ hZ^TG^TPZ^{-1} & 0 & -hR_2 & 0 & 0\\ hG & 0 & 0 & -hR_2^{-1} & 0\\ h(F-L_1C) & 0 & 0 & 0 & -hR_1^{-1}\end{bmatrix} < 0 \quad (6)$$

Pre and post multiplying (6) by diag {I,I,I,I,P} and replacing $-hR_2^{-1}$ by $h(R_2-2I)$ as we know $-R_2^{-1}<(R_2-2I)$, we get

$$\begin{bmatrix}\Omega & hZ^{-T}PGZ & hZ^{-T}PGZ & hG^T & h(F-L_1C)^TP\\ hZ^TG^TPZ^{-1} & -hR_1 & 0 & 0 & 0\\ hZ^TG^TPZ^{-1} & 0 & -hR_2 & 0 & 0\\ hG & 0 & 0 & -hR_2^{-1} & 0\\ hP(F-L_1C) & 0 & 0 & 0 & -hPR_1^{-1}P\end{bmatrix} <0$$

We can replace $-hPR_1^{-1}P$ by $h(R_1-2P)$ as,
$h(R_1-P)\, R_1^{-1}(R_1-P) >0$

$hR_1-hP-hP+ hPR_1^{-1}P>0$

$-hPR_1^{-1}P<h(R_1-2P)$

$$\begin{bmatrix}\Omega & hZ^{-T}PGZ & hZ^{-T}PGZ & hG^T & h(F-L_1C)^TP\\ hZ^TG^TPZ^{-1} & -hR_1 & 0 & 0 & 0\\ hZ^TG^TPZ^{-1} & 0 & -hR_2 & 0 & 0\\ hG & 0 & 0 & h(R_2-2I) & 0\\ hP(F-L_1C) & 0 & 0 & 0 & h(R_1-2P)\end{bmatrix} < 0$$

Now let $PL_1=X$ and defining the matrix (right hand side of the equation) as Σ,

$$\Sigma=\begin{bmatrix} \Omega_1 & hZ^{-T}PGZ & hZ^{-T}PGZ & hG^T & h(F^TP-C^TX^T) \\ hZ^TG^TPZ^{-1} & -hR_1 & 0 & 0 & 0 \\ hZ^TG^TPZ^{-1} & 0 & -hR_2 & 0 & 0 \\ hG & 0 & 0 & h(R_2-2I) & 0 \\ h(PF-XC) & 0 & 0 & 0 & h(R_1-2P) \end{bmatrix}<0$$

Here $\Omega_1=(F^TPZ^{-1}-C^TX^TZ^{-1}+G^TPZ^{-1}+Z^{-T}PF-Z^{-T}XC+Z^{-T}PG)$

For H_∞ observer,it has to satisfy the following equation,

$\int_0^\infty[\dot{V}(e,t)+z(t)^Tz(t)-\gamma^2w(t)^Tw(t)]dt<0$ (7)

If $\dot{V}(e,t)+z(t)^Tz(t)-\gamma^2w(t)^Tw(t)<0$ then (7) will be true also.

$e(t)^T\,\Sigma\,e(t)+e(t)^TC^TCe(t))-\gamma^2w(t)^Tw(t)<0$ [here, $z=y(t)-\hat{y}(t)=Cx(t)-C\hat{x}(t)=Ce(t)$] (8)

if $\boldsymbol{\zeta}(t)=[e(t)\,;\,w(t)]$ and applying Schur complement to (8) ,

$$\boldsymbol{\zeta}(t)^T\begin{bmatrix} \Omega_2 & hZ^{-T}PGZ & hZ^{-T}PGZ & hG^T & h(F^TP-C^TX^T) & 0 \\ hZ^TG^TPZ^{-1} & -hR_1 & 0 & 0 & 0 & 0 \\ hZ^TG^TPZ^{-1} & 0 & -hR_2 & 0 & 0 & 0 \\ hG & 0 & 0 & h(R_2-2I) & 0 & 0 \\ h(PF-XC) & 0 & 0 & 0 & h(R_1-2P) & 0 \\ 0 & 0 & 0 & 0 & 0 & -\gamma^2I \end{bmatrix}\boldsymbol{\zeta}(t)<0$$

where $\Omega_2=(F^TPZ^{-1}-C^TX^TZ^{-1}+G^TPZ^{-1}+Z^{-T}PF-Z^{-T}XC+Z^{-T}PG+C^TC)$

$$\begin{bmatrix} \Omega_2 & hZ^{-T}PGZ & hZ^{-T}PGZ & hG^T & h(F^TP-C^TX^T) & 0 \\ hZ^TG^TPZ^{-1} & -hR_1 & 0 & 0 & 0 & 0 \\ hZ^TG^TPZ^{-1} & 0 & -hR_2 & 0 & 0 & 0 \\ hG & 0 & 0 & h(R_2-2I) & 0 & 0 \\ h(PF-XC) & 0 & 0 & 0 & h(R_1-2P) & 0 \\ 0 & 0 & 0 & 0 & 0 & -\gamma^2I \end{bmatrix}<0$$

According to necessary condition, replacing $F=A$ and $G=A_d$ we get the following final LMI

$$\begin{bmatrix} \Omega_2 & hZ^{-T}PA_dZ & hZ^{-T}PA_dZ & hA_d^T & h(A^TP-C^TX^T) & 0 \\ hZ^TA_d^TPZ^{-1} & -hR_1 & 0 & 0 & 0 & 0 \\ hZ^TA_d^TPZ^{-1} & 0 & -hR_2 & 0 & 0 & 0 \\ hA_d & 0 & 0 & h(R_2-2I) & 0 & 0 \\ h(PA-XC) & 0 & 0 & 0 & h(R_1-2P) & 0 \\ 0 & 0 & 0 & 0 & 0 & -\gamma^2I \end{bmatrix}<0$$

where $\Omega_2=(A^TPZ^{-1}-C^TX^TZ^{-1}+A_d^TPZ^{-1}+Z^{-T}PA-Z^{-T}XC+Z^{-T}PA_d+C^TC)$

Solving the LMI for P and X we can get $L_1=P^{-1}X$.

4. Numerical Example

In this section, we will demonstrate the theory developed in this paper by means of simple examples. Here to solve problem we have used Matlab software,Yalmip Optimization Toolbox and Sedumi solver. Consider the linear continuous time-delay system (9) and (10) with parameters given by

Example (1): $A=\begin{bmatrix}-2 & -0.5\\ 0.5 & -3\end{bmatrix}$ $A_d=\begin{bmatrix}0.5 & 0\\ 0 & 0.5\end{bmatrix}$ (9)

$C=[\ 1 \quad 0\ ]$ $L_2=[0.5 \quad 0.4]^T$ (chosen) (10)

Where $0 < h \leq 0.77$ is an unknown positive scalar.

The purpose is to design H_∞ observer using equation (3) according to the block diagram. The transfer function from exogenous disturbances to error state outputs meets the prescribed H_∞ norm upper bound constraint $\|H_{yw}(s)\|_\infty \leq 0.8$
Here, we take the value γ=0.3

Solving the LMI, we get

$P=\begin{bmatrix}1.0374 & -0.6950\\ -0.6950 & 0.9064\end{bmatrix}$ $R_1=\begin{bmatrix}0.3822 & -0.2531\\ -0.2531 & 0.2936\end{bmatrix}$

$R_2=\begin{bmatrix}0.9766 & -0.3675\\ -0.3675 & 0.8124\end{bmatrix}$ $X=\begin{bmatrix}0.0918\\ 0.2156\end{bmatrix}$

$L_1=\begin{bmatrix}0.5095\\ 0.6285\end{bmatrix}$

Here in the example plant initial state is [5;-2] and estimator initial state is [0;0].

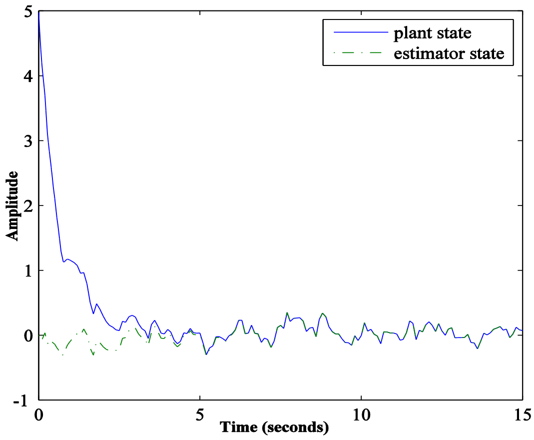

Fig 2. Trajectories of state x_1(t) and $\hat{x}_1$(t)

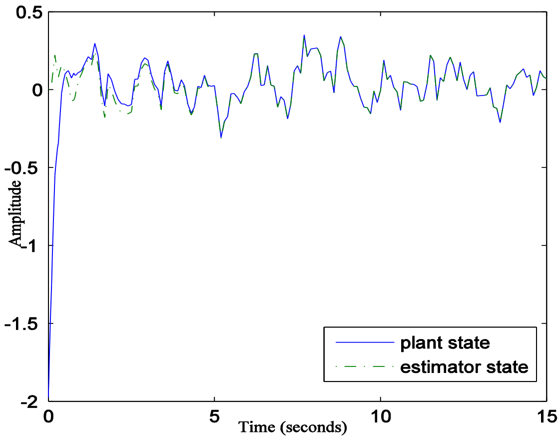

Fig 3. Trajectories of state x_2(t) and $\hat{x}_2$(t)

Example (2):

$A=\begin{bmatrix}-2.5 & 1.2\\ -1.25 & -4.3\end{bmatrix}$ $Ad=\begin{bmatrix}-2.3 & 1.5\\ -1.4 & -3.2\end{bmatrix}$ (11)

$C=[\ 0 \quad 1\ ]$ $L_2=[0.5 \quad 0.4]^T$ (chosen) (12)

Where $0 < h \leq 0.32$ is an unknown positive scalar.

The transfer function from exogenous disturbances to error state outputs meets the prescribed H_∞ norm upper bound constraint $\|H_{yw}(s)\|_\infty \leq 0.8$
Here, we chose γ=0.3.Solving the LMI, we can get the values as follows,

$P=\begin{bmatrix}0.9438 & -0.0715\\ -0.0715 & 0.9943\end{bmatrix}$ $R_1=\begin{bmatrix}0.9319 & -0.2139\\ -0.2139 & 0.9992\end{bmatrix}$

$R_2=\begin{bmatrix}0.8275 & -0.0966\\ -0.0966 & 1.1380\end{bmatrix}$ $X=\begin{bmatrix}2.5462\\ 0.2880\end{bmatrix}$

$L_1=\begin{bmatrix}2.7347\\ 0.4862\end{bmatrix}$

Here in the example plant initial state is [4;-3] and estimator initial state is [0;0**]**.

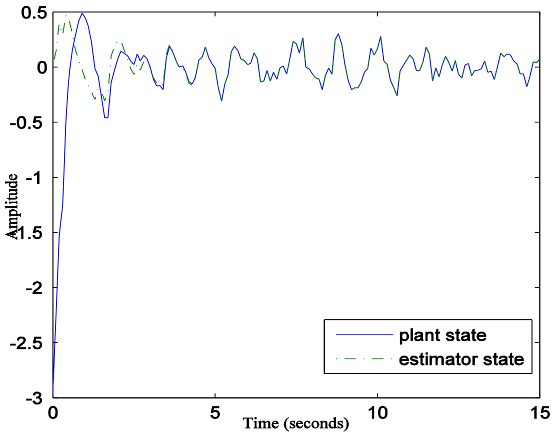

Fig 4. Trajectories of state x_1(t) and $\hat{x}_1$(t)

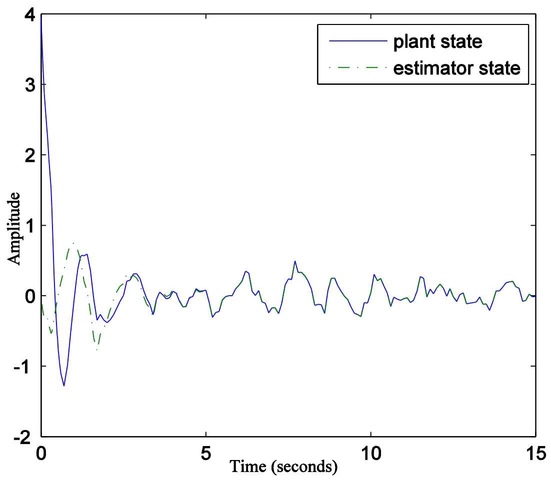

Fig 5. Trajectories of state x_2(t) and $\hat{x}_2$(t)

From the simulation result shown on graphs,we can see that the trajectories of plant states and observer states converge within few seconds,which is pretty good performance by the observer designed using the method developed in this paper.

5. Conclusions

The advantges of such type observer is better estimation of actuall plant states as both state values and rate of change of state values have been taken into consideration in the observer equation.
One of the principle goal for time-dealy systems community is designing observer or controller to achieve longer time delay without interrupting stability.Using the methodology developed in this paper would increase the delay margin.It would be clear in the following comparison table, here same examples are simulated in both Luenberger type observer and proposed observer and obtained delay margin is compared.

Table 1. Comparison table of time delays

Plant	Luenberger type Observer	Proposed Observer
Example 1	0.71 sec	0.77 sec
Example 2	0.28 sec	0.32 sec

It is obvious from the table that when we utilize Luenberger type observer with a system, the system can have states with 0.71 seconds maximum delay (example 1). But utilizing proposed observer, the system can have states with 0.77 seconds maximum delay(example 1). So it means for a system with, lets say 0.75 seconds delay in any of it's state, the Luenberger type observer will not work correctly while the proposed observer will still track down the unknown data. In case of example 2, Luenberger type observer can be used with system having 0.28 seconds maximum delay while proposed observer offer 0.32 seconds delay.

In this paper an observer design procedure for systems with delays in states has been studied. An appropriate gain matrix for observer is calculated while the gain matrix for differentiator block has been predetermined. Necessary and sufficient conditions have also been derived. Numerical examples provided here described the effectiveness of this method.

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Note:

Md. Aminul Haq was born in 1986 in chittagong, Bangladesh. He finished his B.Sc. in Electrical and Electronics Engineering in 2009 and M.Sc. in Control and Automation Engineering in 2016.

Ibrahim Beklan Kucukdemiral received his B.Sc., M.Sc. and Ph.D. degrees from Yildiz Technical University,Department of Electrical Engineering in 1997, 1999, 2002 respectively.He is currently professor of Control and Automation Engineering in Yildiz Technical University.He is a member of several IFAC Technical committee and served as editorial member of different international control congress.His field of research interest is robust,optimal and adaptive control systems, particular focuses are LMI based analysis and synthesis for uncertain TDS, LPV, Convex optimization etc.