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Robust Position Control of a Levitating Ball via a Backstepping Controller

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ABSTRACT

In this paper, a combination of a robust backstepping controller and an integral action for a magnetic levitation system is presented. The mathematical model of the magnetic levitation system containing uncertainties and high-order nonlinear terms has quite a complex structure. The principal aim of this study is to drive the ball position to the desired reference in the presence of a complex structure, parametric uncertainties, and time-varying disturbances. The designed nonlinear controller is based on the robust backstepping technique, in which the robustness is provided via nonlinear damping terms. The boundedness of the tracking error is guaranteed with this method. In order to eliminate steady-state position error caused by the uncertainties and unmodeled dynamics, an integral term is added to the controller structure. After designing the proposed nonlinear controller, the overall closed-loop stability is accordingly analyzed with a Lyapunov-like function. Simulation studies are performed and the results are presented to test the success and the performance of the proposed controller.
***Index Terms*—**Integral action-based control, magnetic levitation system, nonlinear damping, robust backstepping control.

I. INTRODUCTION

Magnetic levitation systems can be defined as systems in which a metal object is moved under the effect of a magnetic field. In these systems, the mechanical contact between moving and fixed parts is removed. Nowadays, major examples demonstrating the application of magnetic levitation systems can be cited as the high-speed trains in Japan and Germany [1], magnetic bearings [2], vibration-isolation systems [3], suspension in air tunnels [4], and planar position systems [5]. Besides these practical applications, in order to conduct academic studies experimentally in a laboratory, a magnetic ball-suspension system has been adopted. In such a system, a ball can be kept suspended by generating the magnetic field through a coil, and the position of the ball can be adjusted by changing the current passing through the coil. The main purpose in this system is to keep the ball stable at a certain position by adjusting the intensity of the magnetic field or to force the ball to track a predefined trajectory.

The magnetic suspension system has a complex behavior due to the highly nonlinear, structured, and unstructured uncertainties in its dynamics, and therefore it is very difficult to obtain the exact model of the system. Another difficulty encountered in the control of magnetic suspension systems is that the force created by the coil changes with the position of the suspended ball and other factors. The fact that the mentioned problem contains uncertainties together with high-order nonlinear terms makes the control problem even more difficult. There are some studies in the literature to obtain an approximate mathematical model of these nonlinear functions existing in the magnetic ball-suspension system [5-8].

The control of the magnetic ball-suspension system has attracted much attention from researchers, mainly because of the aforementioned challenges. In order to ensure stability in case of model uncertainties, many different controller structures have been proposed for magnetic ball-suspension systems and tested in the laboratory environment. In [9], a fuzzy logic-based PID controller is proposed for the nonlinear mathematical model of the magnetic suspension system. A model-free adaptive design approach based on an adaptive-fuzzy procedure versus

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a nonlinear $\mathcal{H}_\infty$ controller has been successfully presented to keep the ball position error near zero [10]. The position of the ball is controlled by combining a neural network for the estimation of electromagnetic parameters with a nonlinear method [11]. For magnetic ball-suspension systems, robust control structures that can deal with parametric uncertainties depending on the mathematical model have been designed with nonlinear damping [12] and internal model-based design [13] techniques. The optimal sliding-mode control structure has been offered in [14] to achieve robust stability as a result of the feedback linearization procedure. Adaptive control [15, 16] and adaptive PID [17] structures realized by designing adaptation rules for uncertain parameters in the system model have also been found in the literature for magnetic ball-suspension systems. In addition to these control structures, a nonlinear predictive control model has been applied to the magnetic ball-suspension system [18]. On the other hand, various structures have been presented in order to predict or observe both disturbance effects in the system and unmeasured state variables [19-22].

The nonlinear backstepping method has been preferred in many applications to control the magnetic suspension system, due to its consistency with the model structure of the system. The main reason for this is the third-order nonlinear dynamics of the system being in the strict-feedback form. A backstepping controller application ignoring the current dynamics is presented in [23]. In the study given by [6], the backstepping method is applied to move the ball to a constant reference. The backstepping control approach is used in combination with other methods in order to deal with model uncertainties and external disturbances. Adaptive backstepping control for model uncertainties [16], and a robust backstepping control algorithm modified to a magnetic ball suspension system are introduced to overcome the aforementioned problems [24]. In addition, auxiliary control rules for the stability analysis of the subsystems in a magnetic ball suspension system have been offered in the proposed algorithm, and the derivatives of these control rules have been obtained with the help of first-order observers in [24]. However, unmatched disturbances may lead to instability in the magnetic levitation system with the proposed controller in [24]. Moreover, to reduce the effect of parametric uncertainties, a backstepping-based nonlinear damping controller with a PI controller has been proposed to eliminate position error [25]. However, the controller is designed for a current-controlled magnetic suspension system utilizing a second-order nonlinear model. On the other hand, a voltage-controlled magnetic levitation system is more challenging in terms of controller design due to the fact that the system order is three. The addition of a cognitive structure to the controller to ensure the robustification of parametric uncertainties [26], and an adaptive backstepping controller design with k-filter robustness modification [27] have been presented for the robust control problem as well. In [28], a robust nonlinear control strategy based on the backstepping approach has been investigated for the control problem, containing high nonlinearities. The robust backstepping control for third-order magnetic suspension system dynamics has been proposed in [29], without considering the elimination of steady-state error caused by constant uncertain parameters in the design procedure.

To the best of the authors' knowledge, the robust control problem has not been considered so far in the backstepping controller design for the complete model of the magnetic suspension system containing parametric uncertainties and time-varying disturbances. Hence, this study proposes a robust backstepping controller with integral action to control the ball position in a magnetic ball-suspension system along a reachable desired position, in the presence of parametric uncertainties and external disturbances. In order to achieve this, the system model is initially simplified by means of a nonlinear transformation. Then, a controller is designed based on a robust backstepping approach, formed by the integral action and the nonlinear damping term, to provide the set point regulation. Almost all of the model constants depending on the physical structure of the system and the gravitational acceleration are considered uncertain. The unknown constant parameters and the external disturbances in the control signal have been bounded by a combination of nonlinear damping and backstepping techniques to meet the stability requirements. In order to eliminate the steady-state error caused by the uncertain parameters, the integral action is contained in the controller and the stability analysis is performed accordingly. Therefore, if the error bounded by the nonlinear damping term is a nonzero constant due to the uncertainties, it can be driven to zero by the integral action. The main contributions of this study are:

- Solving the control problem for a magnetic levitation system by implementing the backstepping technique with an integral action, providing zero steady-state error in case of constant disturbance signal, and
- Revelation of the robustness properties through convergence analysis by means of nonlinear damping terms added in the designed controller.

In conclusion, this paper presents a robust backstepping controller design for the control of the ball position in a magnetic suspension system. The boundedness of the trajectories is provided in the closed-loop system. Moreover, asymptotic stability can be achieved under certain conditions by means of the integral action included in the synthesis part. The proposed controller has been examined via numerical simulations under different conditions in order to test its robustness and performance.

The rest of the paper is organized as follows. In Section II, the dynamic model of the magnetic ball-suspension system is given by introducing structured and unstructured uncertainties. After the robust controller design with nonlinear damping terms based on integral backstepping is devised, the stability of the overall closed-loop system dynamics is shown in the sense of Lyapunov-like function. Then, in following section, the results of the numerical works are presented to show the performance of the proposed controller. Finally, the conclusions are highlighted.

II. MAGNETIC BALL-SUSPENSION SYSTEM

A magnetic field is created by passing a current through a coil in a magnetic ball-suspension system. In this way, a metal ball can be suspended by the generated magnetic field, and the position of the ball can be adjusted with the intensity of the magnetic force. The components of the magnetic ball-suspension system are given in Fig. 1. The dynamic equations for this system can be given as follows:

$$\dot{y} = v + \alpha y \tag{1}$$

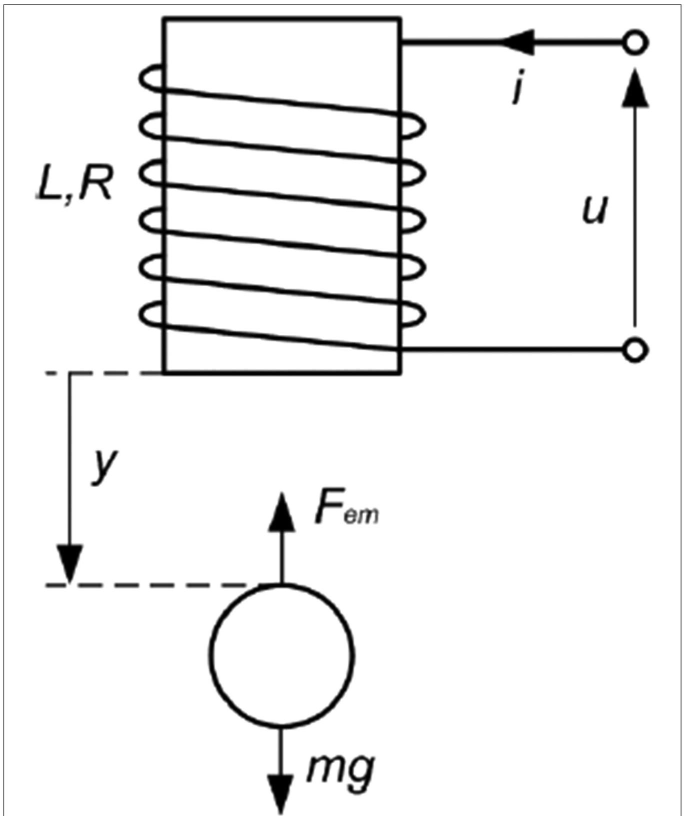

Fig. 1. Magnetic levitation system.

$$\dot{v} = g - \frac{1}{m}F_{em} - \frac{1}{m}d(t) \tag{2}$$

$$\dot{i} = \frac{-R}{L}i + \frac{1}{L}u \tag{3}$$

where y, v, i are the distance between the ball and the coil, the ball's velocity on the vertical axis, and the coil current, respectively. R is the coil resistance, L is the coil inductance, m is the ball's mass, g is the gravitational acceleration, F_{em} is the electromagnetic force generated by the coil, α is the bounded unknown constant, $d(t)$ is the time-varying disturbance term, and u is the voltage input to the coil. Note that, αy contained in (1) can be considered as the position-dependent error in velocity measurement, and the disturbance has a direct effect on the acceleration of the ball corresponding to the magnetic force.

The magnetic force generated by the current through the coil can vary depending on the position of the ball, and this change is not linear. There are different approaches in the literature for obtaining the magnetic force expression [5-8]. In this paper, the force generated by the coil is considered as

$$F_{em} = \frac{ai^2}{(b+y)^2} \tag{4}$$

where a and b denote the positive system constants depending on coil specifications. Details about the derivation of such a force model can be found in [6].

Defining $x_1 = y$, $x_2 = v$, and $x_3 = i^2$, the dynamic model given by (1)–(3) can be transformed to

$$\dot{x}_1 = x_2 + \alpha x_1 \tag{5}$$

$$\dot{x}_2 = \frac{\theta_1}{\theta_2} - \frac{1}{\theta_2}\lambda(x_1)x_3 - \frac{d_\theta(t)}{\theta_2} \tag{6}$$

$$\theta_4\dot{x}_3 = -\theta_3 x_3 + \sqrt{x_3}u \tag{7}$$

with

$$\lambda(x_1) = \frac{1}{(x_1+b)^2} \tag{8}$$

where $\theta_1 = g\theta_2$, $\theta_2 = \frac{m}{a}$, $d_\theta(t) = \frac{\theta_2 d(t)}{m}$, $\theta_3 = R$, $\theta_4 = \frac{L}{2}$. Here, $d_\theta(t)$ is the bounded disturbance term and it satisfies

$$|d_\theta(t)| \le \bar{d}_\theta \tag{9}$$

where $\bar{d}_\theta$ is a positive upper bound.

The main purpose of this study is to regulate the position of the ball at a reference point despite the uncertain parameters $\theta_i, i = \{1, 2, 3, 4\}$ and the external disturbance $d_\theta(t)$.

III. CONTROLLER DESIGN AND STABILITY ANALYSIS

In this section, a robust backstepping controller design procedure is presented. The structure of the controller is convenient for the control of the ball position in a magnetic levitation system, but it is not able to eliminate the steady-state error caused by the parametric uncertainties and/or external disturbances. For this reason, the proposed controller is supported by the integral action in order to guarantee high-precision and high-performance position control.

Let the desired set point for the motion of the ball be x_{1d}. Then, the desired trajectory for the velocity is given by $x_{2d} = \dot{x}_{1d} = 0$. Accordingly, the desired reference values and their time derivatives are zero except for x_{1d} through all controller design steps. Error signals for the position and velocity of the ball on the vertical axis can be generated using the desired trajectory signals as follows:

$$e_1 = x_1 - x_{1d} \tag{10}$$

$$e_2 = x_2. \tag{11}$$

Utilizing the system dynamics (5)–(7) and (10)–(11), the error dynamics can be obtained as

$$\dot{e}_1 = e_2 + \alpha x_1 \tag{12}$$

$$\dot{e}_2 = \frac{\theta_1}{\theta_2} - \frac{1}{\theta_2}\lambda(x_1)x_3 - \frac{d_\theta(t)}{\theta_2} \tag{13}$$

$$\theta_4\dot{x}_3 = -\theta_3 x_3 + \sqrt{x_3}u. \tag{14}$$

Let the error integral term be introduced by

$$e_0 = \int_0^t e_1(t)dt - \frac{\alpha}{k_0} x_{1d} \tag{15}$$

with $k_0 \in \mathbb{R}^+$ and consider

$$V_1 = \begin{bmatrix} e_0 & e_1 \end{bmatrix} P \begin{bmatrix} e_0 \\ e_1 \end{bmatrix} \tag{16}$$

where P is a positive definite matrix given by

$$P = \frac{1}{2}\begin{bmatrix} k_0 + k_1 - \alpha & 1 \\ 1 & 1 \end{bmatrix} \tag{17}$$

with k_1 being a positive constant satisfying $k_1 > \alpha + 1$. Employing

$$\phi_2 = -k_0 \int_0^t e_1(t)dt - k_1 e_1 \tag{18}$$

where $k_1 \in \mathbb{R}^+$ and

$$z_2 = e_2 - \phi_2, \tag{19}$$

the time derivative of (16) can be obtained as

$$\dot{V}_1 = -k_0 e_0^2 - (k_1 - \alpha - 1)e_1^2 + (e_0 + e_1)z_2. \tag{20}$$

And the error dynamics given by (12) can be rearranged as follows

$$\dot{e}_1 = -k_0 e_0 - (k_1 - \alpha)e_1 + z_2. \tag{21}$$

Note that V_1 in (16) is positive definite in terms of e_0 and e_1, and its time derivative is negative definite if $z_2 = 0$ and $k_1 - \alpha - 1 < 0$. Thus, $e_0 \to 0$ and $e_1 \to 0$ are provided as $t \to \infty$. Utilizing $\dot{e}_2$ and $\dot{\phi}_2$, the derivative of (19) with respect to time can be organized as

$$\theta_2 \dot{z}_2 = \theta_1 + \theta_2 \rho_1 - \lambda x_3 - d_\theta + \theta_5 k_1 x_1 \tag{22}$$

where

$$\rho_1 = (k_0 - k_1^2)e_1 + k_1 z_2 - k_0 k_1 \int_0^t e_1(t)dt \tag{23}$$

and $\theta_5 = \alpha\theta_2$.

x_3 signal will be used in the next step of the backstepping controller design. However, there are unknown constants in z_2 dynamics. Here, the system uncertainties do not directly match the control signal. In order to overcome these uncertainties, the unknown constant parameters in the control signal will be bounded by combining the nonlinear damping and backstepping control technique. The second auxiliary function, V_2, can be given by

$$V_2 = V_1 + \frac{1}{2}\theta_2 z_2^2, \tag{24}$$

and the time derivative of this function can be obtained as

$$\begin{aligned} \dot{V}_2 = &-k_0 e_0^2 - (k_1 - \alpha - 1)e_1^2 + z_2(e_0 + e_1 + \theta_1 \\ &+ \theta_2 \rho_1 - \lambda x_3 - d_\theta + \theta_5 k_1 x_1). \end{aligned} \tag{25}$$

At this point, assigning

$$\begin{aligned} \phi_3 = &\frac{1}{\lambda}(k_2 z_2 + (\kappa_\alpha + \kappa_1 + \kappa_d)z_2 + \kappa_2 \rho_1^2 z_2 \\ &+ \kappa_5 k_1^2 x_1^2 z_2 + \int_0^t e_1(t)dt + e_1) \end{aligned} \tag{26}$$

and

$$z_3 = x_3 - \phi_3 \tag{27}$$

where $k_2, \kappa_1, \kappa_2, \kappa_5, \kappa_\alpha, \kappa_d \in \mathbb{R}^+$, one can reorganize (25) to be

$$\begin{aligned} \dot{V}_2 = &-k_0 e_0^2 - (k_1 - \alpha - 1)e_1^2 - k_2 z_2^2 + z_2(-\theta_\alpha + \theta_1 \\ &+ \theta_2 \rho_1 - \lambda z_3 - d_\theta + \theta_5 k_1 x_1 - (\kappa_\alpha + \kappa_1 + \kappa_d)z_2 \\ &- \kappa_2 \rho_1^2 z_2 - \kappa_5 k_1^2 x_1^2 z_2). \end{aligned} \tag{28}$$

with $\theta_\alpha = \frac{\alpha}{k_0} x_{1d}$. For the last step of the design procedure, the derivative of the (27) can be formed as

$$\theta_4 \dot{z}_3 = -\theta_3 x_3 - \theta_4 \dot{\phi}_3 + \sqrt{x_3}u \tag{29}$$

after the utilization of (14). $\theta_4 \dot{\phi}_3$ in that equation is quite complex and it can be derived as

$$\begin{aligned} \theta_4 \dot{\phi}_3 = &-\theta_4 \frac{\dot{\lambda}}{\lambda^2}\rho_2 + \theta_4 \frac{1}{\lambda}\rho_3 + \frac{1}{\lambda}\rho_4(\theta_6 - \theta_7 \lambda x_3 \\ &+ \theta_8 k_1 x_1 - h_\theta + \theta_4 \rho_1) + \theta_9 \frac{1}{\lambda}\rho_5 \end{aligned} \tag{30}$$

where

$$\begin{aligned} \rho_2 = &(k_2 + \kappa_\alpha + \kappa_1 + \kappa_d + \kappa_2 \rho_1^2 + \kappa_5 k_1^2 x_1^2)z_2 + e_1 \\ &+ \int_0^t e_1(t)dt \end{aligned} \tag{31}$$

$$\rho_3 = \left(1 + 2\kappa_2 \rho_1 z_2 (k_0 - k_1^2)\right)(z_2 - k_1 e_1 - k_0 \int_0^t e_1(t)dt) \tag{32}$$

$$\rho_4 = k_1 + k_2 + \kappa_1 + \kappa_d + \kappa_2 \rho_1^2 + k_1^2 \kappa_5 x_1^2 \tag{33}$$

$$\rho_5 = (1 + 2\kappa_2 \rho_1 z_2 (k_0 - k_1^2) + 2k_1^2 \kappa_5 z_2)x_1 \tag{34}$$

and $\theta_6 = \frac{\theta_1 \theta_4}{\theta_2}$, $\theta_7 = \frac{\theta_4}{\theta_2}$, $\theta_8 = \frac{\theta_4 \theta_5}{\theta_2}$, $\theta_9 = \alpha\theta_4$ and $h_\theta = \frac{\theta_4 d_\theta}{\theta_2}$. Note that, h_θ which includes both constant uncertainties and time-varying disturbance, is bounded and it satisfies

$$|h_\theta(t)| \le \bar{h}_\theta \tag{35}$$

where $\bar{h}_\theta$ is a positive constant. After these mathematical manipulations, the closed-loop dynamics turns out to be

$$\dot{e}_0 = e_1 \tag{36}$$

$$\dot{e}_1 = (\alpha - k_1)e_1 - k_0 e_0 + z_2 \tag{37}$$

$$\theta_2 \dot{z}_2 = \theta_1 + \theta_2 \rho_1 - \lambda x_3 - d_\theta + k_1 \theta_5 x_1 \tag{38}$$

$$\theta_4 \dot{z}_3 = -\theta_3 x_3 - \theta_4 \dot{\phi}_3 + \sqrt{x_3} u. \tag{39}$$

The candidate Lyapunov function to construct the control signal making the closed-loop system globally bounded can be introduced as

$$V = V_2 + \frac{1}{2}\theta_4 z_3^2. \tag{40}$$

Utilizing (36)–(39), the time derivative of this function can be obtained as

$$\begin{aligned} \dot{V} = & -k_0 e_0^2 - (k_1 - \alpha - 1)e_1^2 + z_2(e_0 + e_1 + \theta_1 + \theta_2 \rho_1 \\ & -\lambda x_3 - d_\theta + k_1 \theta_5 x_1) + z_3(-\theta_3 x_3 - \theta_4 \dot{\phi}_3 + \sqrt{x_3} u). \end{aligned} \tag{41}$$

This expression can be reorganized by plugging in ϕ_3 and $\theta_4 \dot{\phi}_3$ as

$$\begin{aligned} \dot{V} = & -k_0 e_0^2 - (k_1 - \alpha - 1)e_1^2 - k_2 z_2^2 + z_2(-\theta_\alpha + \theta_1 \\ & + \theta_2 \rho_1 - d_\theta + \theta_5 k_1 x_1 - (\kappa_\alpha + \kappa_1 + \kappa_d) z_2 - \kappa_2 \rho_1^2 z_2 \\ & - \kappa_5 k_1^2 x_1^2 z_2) + z_3 \Big[-\lambda z_2 + \sqrt{x_3} u - \theta_3 x_3 \\ & + \theta_4 \left(\frac{\dot{\lambda}}{\lambda^2} \rho_2 - \frac{1}{\lambda} \rho_3 - \frac{1}{\lambda} \rho_1 \rho_4 \right) - \theta_6 \frac{1}{\lambda} \rho_4 + \theta_7 x_3 \rho_4 \\ & - \theta_8 \frac{1}{\lambda} k_1 x_1 \rho_4 + h_\theta \frac{1}{\lambda} \rho_4 - \theta_9 \frac{1}{\lambda} \rho_5 \Big]. \end{aligned} \tag{42}$$

Finally, the control signal can be formed as

$$u = \frac{1}{\sqrt{x_3}} \left(\lambda z_2 - k_3 z_3 + \psi \right) \tag{43}$$

where $k_3 \in \mathbb{R}^+$ and ψ are a design terms to be introduced. Employing this control signal, the derivative of the candidate Lyapunov function turns into

$$\begin{aligned} \dot{V} = & -k_0 e_0^2 - (k_1 - \alpha - 1)e_1^2 - k_2 z_2^2 - \theta_\alpha z_2 + \theta_1 z_2 \\ & + \theta_2 \rho_1 z_2 - d_\theta z_2 + \theta_5 k_1 x_1 z_2 - (\kappa_\alpha + \kappa_1 + \kappa_d \\ & - \kappa_2 \rho_1^2) z_2^2 - k_3 z_3^2 + \psi z_3 - \theta_3 x_3 z_3 - \theta_6 \frac{1}{\lambda} \rho_4 z_3 \\ & + \theta_4 \left(\frac{\dot{\lambda}}{\lambda^2} \rho_2 - \frac{1}{\lambda} \rho_3 - \frac{1}{\lambda} \rho_1 \rho_4 \right) z_3 + \theta_7 x_3 \rho_4 z_3 \\ & - \theta_8 \frac{1}{\lambda} k_1 x_1 \rho_4 z_3 + h_\theta \frac{1}{\lambda} \rho_4 z_3 - \theta_9 \frac{1}{\lambda} \rho_5 z_3. \end{aligned} \tag{44}$$

Moreover, ψ can be assigned as

$$\begin{aligned} \psi = -z_3 \Bigg[& \kappa_3 x_3^2 + \kappa_4 \left(\frac{\dot{\lambda}}{\lambda^2} \rho_2 - \frac{1}{\lambda} \rho_3 - \frac{1}{\lambda} \rho_1 \rho_4 \right)^2 \\ & + \frac{\kappa_6 \rho_4^2}{\lambda^2} + \kappa_7 x_3^2 \rho_4^2 + \frac{\kappa_h \rho_4^2}{\lambda^2} + \frac{\kappa_8 k_1^2 x_1^2 \rho_4^2}{\lambda^2} + \frac{\kappa_9 \rho_5^2}{\lambda^2} \Bigg] \end{aligned} \tag{45}$$

with positive constants $\kappa_i, i \in \{3, 4, 6, 7, 8, h\}$. Adding and subtracting $\frac{1}{4}\sum_{i=1}^{8} \frac{\theta_i^2}{\kappa_i}$, $\frac{\theta_\alpha^2}{\kappa_\alpha}$, $\frac{d_\theta^2}{4\kappa_d}$, and $\frac{h_\theta^2}{4\kappa_h}$ to/from (44) becomes

$$\begin{aligned} \dot{V} = & -k_0 e_0^2 - (k_1 - \alpha - 1)e_1^2 - k_2 z_2^2 - \lambda z_2 z_3 \\ & - \kappa_\alpha \left(z_2 - \frac{\theta_\alpha}{2\kappa_\alpha} \right)^2 - \kappa_1 \left(z_2 - \frac{\theta_1}{2\kappa_1} \right)^2 \\ & - \kappa_2 \left(\rho_1 z_2 - \frac{\theta_2}{2\kappa_2} \right)^2 - \kappa_3 \left(x_3 z_3 - \frac{\theta_3}{2\kappa_3} \right)^2 \\ & - \kappa_4 \left(\left(\frac{\dot{\lambda}}{\lambda^2} \rho_2 - \frac{1}{\lambda} \rho_3 - \frac{1}{\lambda} \rho_1 \rho_4 \right) z_3 - \frac{\theta_2}{2\kappa_4} \right)^2 \\ & - \kappa_5 \left(k_1 x_1 z_2 - \frac{\theta_5}{2\kappa_5} \right)^2 - \kappa_6 \left(\frac{1}{\lambda} \rho_4 z_3 - \frac{\theta_6}{2\kappa_6} \right)^2 \\ & - \kappa_7 \left(x_3 \rho_4 z_3 - \frac{\theta_7}{2\kappa_7} \right)^2 - \kappa_8 \left(\frac{1}{\lambda} k_1 x_1 \rho_4 z_3 - \frac{\theta_8}{2\kappa_8} \right)^2 \\ & - \kappa_9 \left(\frac{1}{\lambda} \rho_5 - \frac{\theta_9}{2\kappa_9} \right)^2 - \kappa_h \left(\frac{1}{\lambda} \rho_4 z_3 - \frac{h_\theta}{2\kappa_h} \right)^2 \\ & - \kappa_d \left(z_2 - \frac{d_\theta}{2\kappa_d} \right)^2 + \frac{1}{4}\sum_{i=1}^{9} \frac{\theta_i^2}{\kappa_i} + \frac{\theta_\alpha^2}{4\kappa_\alpha} + \frac{d_\theta^2}{4\kappa_d} + \frac{h_\theta^2}{4\kappa_h} \end{aligned} \tag{46}$$

satisfying

$$\begin{aligned} \dot{V} \le & -k_0 e_0^2 - (k_1 - \alpha - 1)e_1^2 - k_2 z_2^2 - k_3 z_3^2 \\ & + \frac{1}{4}\sum_{i=1}^{9} \frac{\theta_i^2}{\kappa_i} + \frac{\theta_\alpha^2}{4\kappa_\alpha} + \frac{d_\theta^2}{4\kappa_d} + \frac{h_\theta^2}{4\kappa_h} \\ \le & -k_0 e_0^2 - (k_1 - \alpha - 1)e_1^2 - k_2 z_2^2 - k_3 z_3^2 \\ & + \frac{1}{4}\sum_{i=1}^{9} \frac{\theta_i^2}{\kappa_i} + \frac{\theta_\alpha^2}{4\kappa_\alpha} + \frac{\bar{d}_\theta^2}{4\kappa_d} + \frac{\bar{h}_\theta^2}{4\kappa_h} \end{aligned} \tag{47}$$

Since $\left(\frac{1}{4}\sum_{i=1}^{9} \frac{\theta_i^2}{\kappa_i} + \frac{\theta_\alpha^2}{4\kappa_\alpha} + \frac{\bar{d}_\theta^2}{4\kappa_d} + \frac{\bar{h}_\theta^2}{4\kappa_h} \right)$ is bounded, it follows that if $\alpha < k_1$ and $\left| (\alpha - k_1)e_1^2 - k_2 z_2^2 - k_3 z_3^2 \right|$ is larger than $\left(\frac{1}{4}\sum_{i=1}^{9} \frac{\theta_i^2}{\kappa_i} + \frac{\theta_\alpha^2}{4\kappa_\alpha} + \frac{\bar{d}_\theta^2}{4\kappa_d} + \frac{\bar{h}_\theta^2}{4\kappa_h} \right)$, then $\dot{V} < 0$ is satisfied. Thus, considering (40) and (47), e_0, e_1, z_2, and z_3 are guaranteed to be globally ultimately bounded [30]. Therefore, e_2 and x_3 are globally ultimately bounded as well, according to (19) and (27). The ball position error is ultimately bounded, and the ultimate bound can be tuned by changing the parameters ($\kappa_i, i = \{1, \ldots, 9, \alpha, d, h\}$) and the controller gains ($\kappa_j, j \in \{1, 2, 3\}$).

TABLE. PHYSICAL PARAMETERS OF THE DYNAMIC MODEL

Model Parameter	Value
m	0.005 kg
R	22 Ω
L	0.5 H
g	9.81 m/s^2
a	0.003
b	0.041

Also, in order to eliminate the steady-state error of the ball position, the integral action, which is well-known in control theory, is added from the beginning to the conventional robust backstepping controller based on nonlinear damping terms. The integral action guarantees convergence of e_1 to zero in the presence of time invariant steady-state error.

IV. SIMULATION RESULTS

In order to test the performance and robustness of the presented controller, numerical simulations have been carried out. The physical parameters of the magnetic ball-suspension system used in the simulations are presented in Table. In addition, the input signal (u) is allowed to take a value in the interval of 0–30 V. The initial values of the state variables have been assigned as $\begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}^T = \begin{bmatrix} 0.02 & 0 & 0.001 \end{bmatrix}^T$. Controller gains have been taken as $k_1 = 50, k_2 = 2000, k_3 = 500, \kappa_i = 0.1, i = \{1, \ldots, 8\}, \kappa_9 = 0.5 \times 10^{-3}$, $\kappa_\alpha = 0.01$, $\kappa_d = 0.01$, $\kappa_h = 0.01$, and integral gain has been assigned as $k_i = 0.01$. The solver step time for the model has been set to 1 µs and the controller sampling time to 100 µs.

In the numerical simulations, the bounded unknown constant (α) has been set to 0.01. The simulation results are presented for two different disturbance signals. External disturbance is added as $d(t) = 10$ and $d(t) = 10 \sin(2\pi t)$, respectively. Note that the disturbance signal, which is caused by external unstructured dynamics, is a bounded function but not an exponentially decaying disturbance signal in the second case. Note also that the disturbance signal has a direct effect on the ball acceleration, hence it can be considered as a disturbance on the force applied to the ball.

A square wave signal has been used for the desired ball position in the numerical simulations. The reference input has initially been assigned as $x_{1d} = 0.02$ m, then changed between 0.02 m and 0.025 m to better analyze the step response. The change of the ball position and velocity, and the change of coil current and input voltage are presented in Fig. 2 and 3 for different disturbance signals. Since the coil current and accordingly the magnetic field force are very close to zero at the beginning of the simulation, it should be noticed that the ball moves in the direction of gravity. On the other hand, the ball position approaches the desired value rapidly with the generation of the magnetic force. When the results are examined, it can be observed that the setting time is approximately 0.25 s, and the percentage overshoot is approximately 20%.

As a result, the control of the ball position is successfully achieved for set-point reference. The proposed robust backstepping controller with integral action drives the error to zero in the presence of

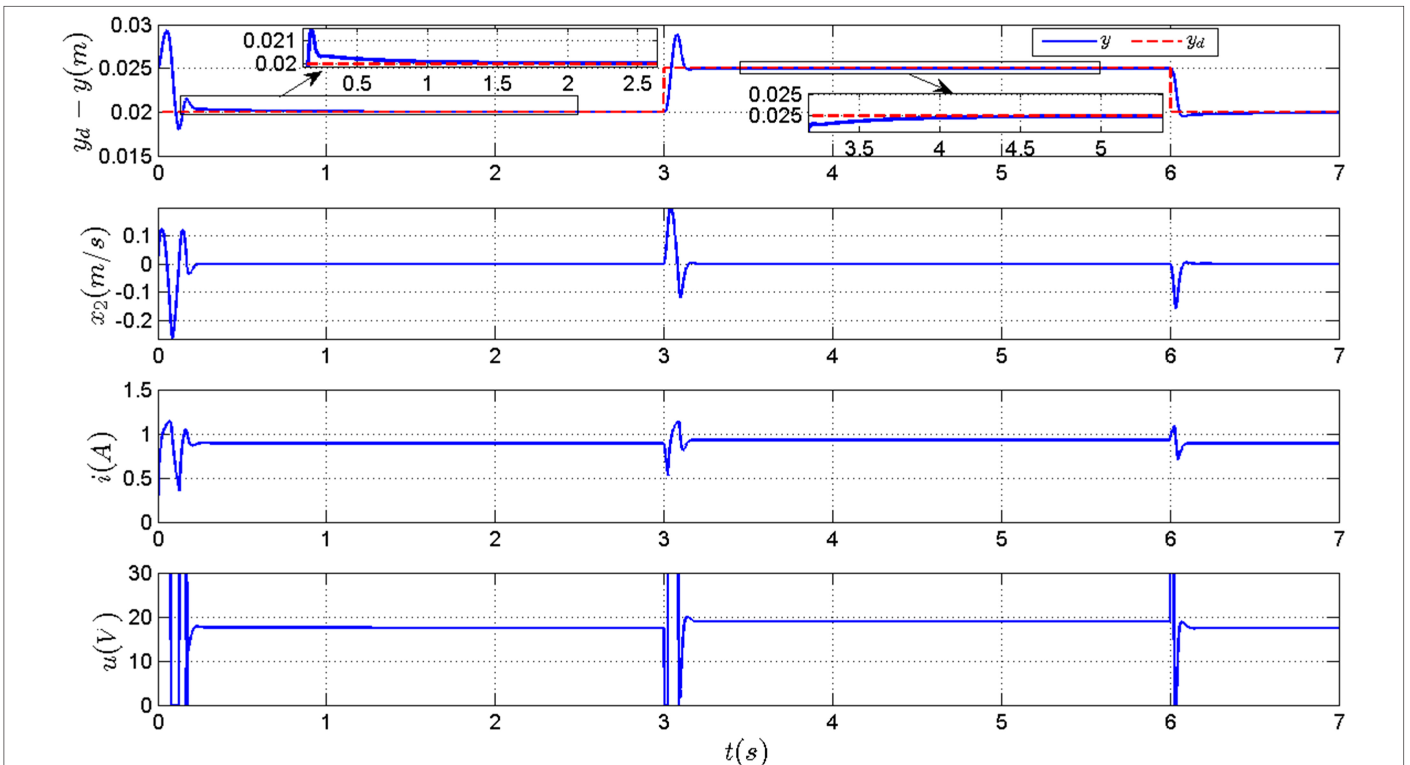

Fig. 2. Response of ball position and velocity, coil current, and voltage for fixed-point reference.

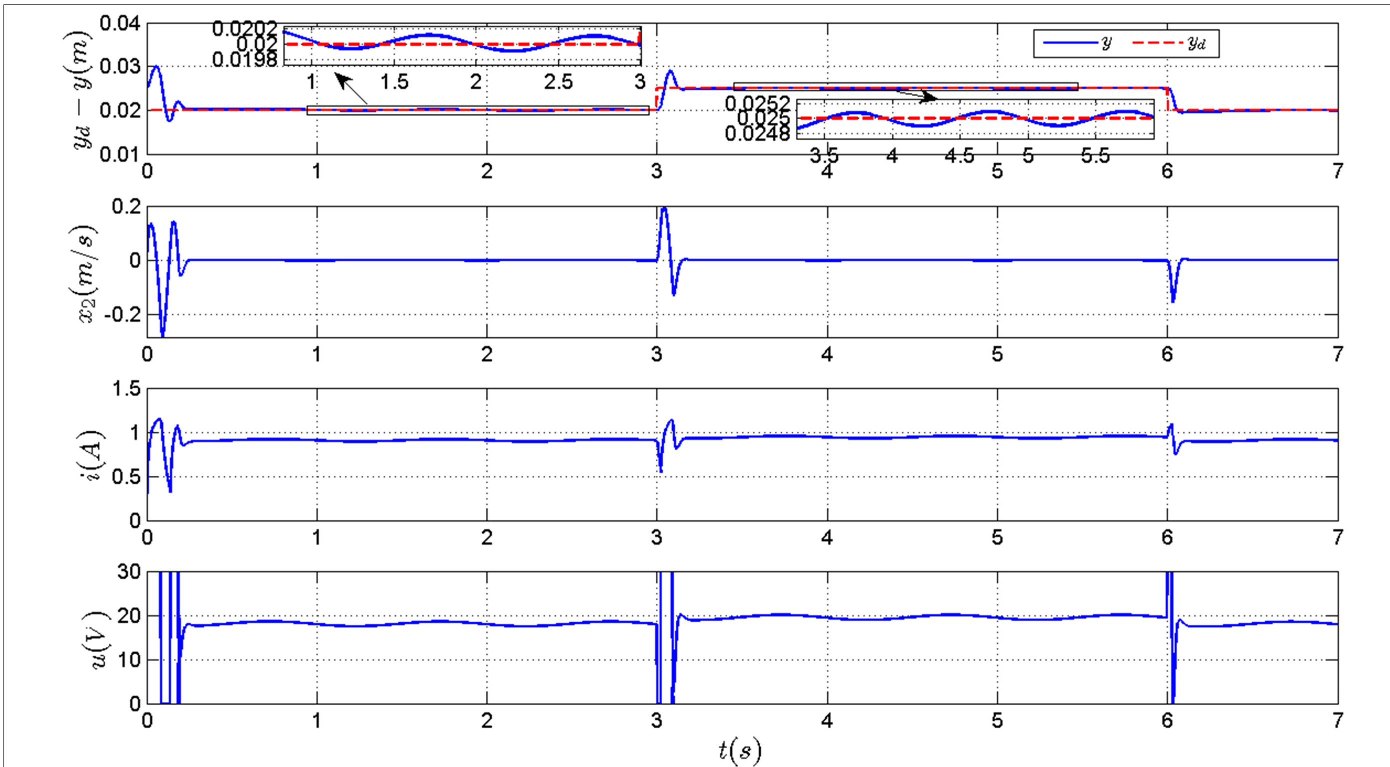

Fig. 3. Response of ball position and velocity, coil current, and voltage for fixed-point reference.

constant external disturbance signal and parametric uncertainties (Fig. 2). In case of the time-varying disturbance signal, the proposed controller drives the position error to almost zero (Fig. 3). In addition, the proposed controller neither has any information about uncertain parameters and disturbance, nor their bounded values. Consequently, the effectiveness of the proposed controller has been shown by means of a numerical simulation and by the theoretical analysis provided in the design step.

V. CONCLUSIONS

In this study, a robust backstepping controller with integral action has been designed and presented to ensure that the ball position is driven to a reference in a magnetic ball-suspension system. After obtaining the error dynamics for the complete model of a magnetic levitation system and designing the non-linear damping-based robust backstepping controller with integral action, the boundedness of the error signal in the closed-loop dynamics with the proposed controller structure has been shown in the sense of a Lyapunov-like analysis. With the inclusion of the integral action in the robust backstepping control methodology, the convergence of the position error of the ball in a bounded region has been guaranteed and the analysis has been carried out accordingly. Subsequently, the designed controller has been implemented via a numerical simulation run for a magnetic levitation system and satisfactory results have been obtained.

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