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Superdense Coding, Teleportation Algorithms, and Bell's Inequality Test in Qiskit and IBM Circuit Composer

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ABSTRACT

Quantum teleportation is a technique of sending information from one place to another place. Distance between two points can be hundreds of thousands of light-years. For quantum teleportation, there is no need for a channel between two points when sending a state vector from one place to another. Since classical information sharing is possible, it is also possible to send a state vector from one place to another place. Teleportation is the transfer of a quantum state from one place to another through classical channels. Superdense coding, a dual to teleportation, uses a single quantum bit to transmit two bits classical information. Superdense coding uses a qubit to transfer two classical bits, while teleportation performs one qubit transfer using two classical bits. In this article, teleportation, superdense coding algorithms, and the Bell's inequality test in which Bell's inequality is violated with quantum mechanics are performed on both Qiskit and International Business Machines circuit composer, and results are compared and presented in detail. The results revealed that whether a faster-than-light signal transfer is possible using quantum mechanics depends on whether a copy of the quantum state is created or not. Finally, Bell's inequality created by classical logic violated by quantum mechanics is shown by experimental results.

***Index Terms*—**Superdense coding, quantum teleportation, Bell's inequality

I. INTRODUCTION

Quantum information and quantum computing perform calculations exploiting the properties of quantum states, such as superposition, interference, entanglement, and fundamental quantum mechanical system to solve certain problems. The devices built for the purpose of performing quantum computations are known as quantum computers. Today, International Business Machines (IBM), Microsoft, Google, Rigetti, Honeywell, and so many companies in China are trying to build quantum computers to reach the end of perfectly working quantum computers. Recently, IBM has unveiled the state-of-the-art quantum computer in Germany and thus Germany has been included in the quantum world as well. To solve certain problems, significant improvements over classical computing has been realized with quantum computing by exploiting the quantum computers developed by these companies and countries so far. Berstain-Vazirani, Deutsch and Deutsch-Jozsa, Shor's algorithm, Grover's algorithm, quantum Fourier transformation, and quantum phase estimation are examples of the well-known improvements in quantum computation world. Along with these developments, the question arose whether it would be possible to transfer information from one place to another faster than light by using quantum mechanical systems. The answer to this question depends on whether it is possible to copy a quantum state [1, 2].

A quantum mechanical system is difficult to understand as it often violates classical logic. Quantum mechanical states are very delicate, delicate states that can deteriorate at the slightest intervention. Due to various physical reasons, it is not possible to copy a state vector from one place to another. However, according to the rules of the quantum mechanical systems, moving is possible, and copying is impossible. This situation is the so-called no-cloning theorem. Despite all these physical impossibilities, quantum developers started to work with the aim of sending information from one place to another. In this way, they have developed algorithms so-called superdense coding and teleportation, which performs the transfer of classical information and

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quantum state, respectively. First, the teleportation algorithm was developed in 1993 and experimentally verified in 1997 by Bennett, Brassard, and other developers [3, 4]. The superdense coding algorithm was initially developed by Bennett and Wiessner, then superdence coding transmission was experimentally demonstrated by Anton Zeillinger [5, 6], and finally, it was specified as a secure communication protocol [7]. While superdense coding uses a single qubit to send two classical bits information, teleportation uses two classical bits information to send a single qubit.

In recent years, a great deal of implementation concerning with the teleportation and superdense coding algorithms has been proposed. In [8], a scheme to obtain superdense coding for the quadrature amplitude values of the electromagnetic field is demonstrated. In the protocol, shared entanglement provided by nondegenerate parametric down conversion in the limit of large gain is utilized to attain high efficiency. In [9], the proof of principle demonstration of five-photon entanglement and open destination teleportation is presented. For the purpose of getting experimental results, two entangled photon pairs to generate a four-photon entangled state are used, which is then combined with a single photon state to achieve the goals. In [10], unconditional teleportation between diamond spin qubits residing in independent setups separated by 3 m are presented. Result is obtained by fully separating the generation of remote entanglement from the two-qubit Bell-state measurement and feed-forward. State-of-the-art approach for teleportation and superdense coding algorithms is presented in [11], the purpose is to give wide perspective to presented solutions for quantum circuit mapping by exploiting quantum teleportation. For this purpose, quantum teleportation is feasible for quantum networks has been demonstrated.

Besides teleportation amd superdense coding algorithms, Bell's inequality test is one of the most intriguing circuits for quantum devices. Bell's inequality is an inequality created entirely by classical logic, which excludes the quantum mechanical system introduced by the famous physicist John Bell. Bell deduced that if measurements are performed independently on the two separated halves of a pair, then the assumption that the outcomes depend upon hidden variables within each half implies a constraint on how the outcomes on the two halves are correlated. Bell's inequality test concern measurements made by observers on pairs of particles that have interacted and then separated [12, 13]. Bell's inequality which is violated by the quantum mechanical system has been revealed by many studies. In [14], Bell's inequality between the quantum states of two remote Yb+ ions whose distance is about 1 m is violated. The entanglement of two ions has been built by interference and joint detection of two emitted photons and is characterized by full quantum state tomography.

This study deals with superdense coding, teleportation, and Bell's inequality test. The success rates of the algorithms are implemented both in the simulation environment using Qiskit and on real quantum devices, and detailed results are presented. For test processes on real quantum device, IBM's 32 qubits or so-called ibm_qasm_simulator devices are chosen.

The rest of the paper is organized as follows: in Section 2, the superdense coding, teleportation, and Bell's inequality are explained; in Section 3, the experimental results are presented and comparisons are made, and finally in Section 4, the conclusions are given.

II. QUANTUM TELEPORTATION APPLICATIONS

A. Superdense Coding Algorithm

Superdense coding is one of the interesting applications of quantum entanglement. To implement superdense coding, it is assumed that there are two people named Alice and Bob. Alice wants to send a two-bit string of classical information to Bob. These two bits of classical information have special messages. There are four possible combinations of these two bits of the classical information. Alice wants to send any of these two bits of classical information but there is one rule. Alice is only allowed to send one qubit of information to Bob. To share two bits of classical information, they have to share an entangled state among themselves in advance. This situation is known as superdense coding protocol. Quantum circuit design of the superdense coding algorithm is shown in Fig. 1.

Eve prepares two qubits in the entangled state $|\Psi_0>$ as seen in Eq. (1) and Alice and Bob share a pair of qubits:

$$|\Psi_0> = |00> \quad (1)$$

First, Alice applies the Hadamard gate to the first qubit and obtains the $|\Psi_1>$ state as seen in Eq. (2):

$$|\psi_1\rangle = \frac{|00\rangle + |10\rangle}{\sqrt{2}} \quad (2)$$

Then CNOT gate is applied to both of first and second qubit as source and target, respectively, and $|\Psi_2>$ is obtained as seen in Eq. (3):

$$|\psi_2\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}} \quad (3)$$

If Alice wants to send 00 classic information, I operator should be used. I operator does not change anything on $|\Psi_2>$ state and as seen in Eq. (4), the situation maintains:

$$|\psi_3\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}} \quad (4)$$

When CNOT gate is applied to $|\Psi_3>$, $|\Psi_4>$ state is obtained as seen in Eq. (5):

$$|\psi_4\rangle = \frac{|00\rangle + |10\rangle}{\sqrt{2}} \quad (5)$$

Finally, when Hadamard gate is applied, $|\Psi_5>$ state is obtained as seen in Eq. (6):

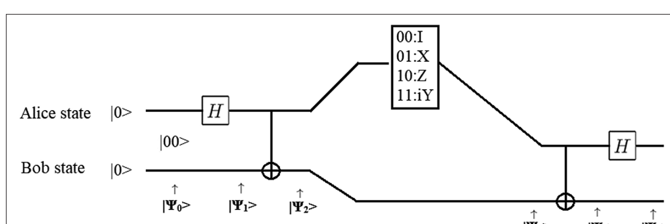

Fig. 1. Quantum circuit design of superdense coding algorithm.

$$\left|\psi_5\right\rangle = \frac{1}{2}\left(\left|00\right\rangle + \left|10\right\rangle + \left|00\right\rangle - \left|10\right\rangle\right) = \left|00\right\rangle \quad (6)$$

When the obtained value is examined, it is seen that the 00-information string is obtained with 100% probability.

If Alice wants to send 01 classic information, X operator is used and obtained state is $|\Psi_3>$ as seen in Eq. (7):

$$\left|\psi_3\right\rangle = \frac{\left|10\right\rangle + \left|01\right\rangle}{\sqrt{2}} \quad (7)$$

When the CNOT gate is applied, $|\Psi_4>$ state is obtained as seen in Eq. (8):

$$\left|\psi_4\right\rangle = \frac{\left|11\right\rangle + \left|01\right\rangle}{\sqrt{2}} \quad (8)$$

Finally, when Hadamard gate is applied, $|\Psi_5>$ state is obtained as seen in Eq. (9):

$$\left|\psi_5\right\rangle = \frac{1}{2}\left(\left|01\right\rangle - \left|11\right\rangle + \left|01\right\rangle + \left|11\right\rangle\right) = \left|01\right\rangle \quad (9)$$

When the obtained value is examined, it is seen that 01-information string is obtained with 100% probability.

If Alice wants to send 10 classic information, Z operator is used and obtained state is $|\Psi_3>$ as seen in Eq. (10):

$$\left|\psi_3\right\rangle = \frac{\left|00\right\rangle - \left|11\right\rangle}{\sqrt{2}} \quad (10)$$

When CNOT gate is applied, $|\Psi_4>$ state is obtained as seen in Eq. (11):

$$\left|\psi_4\right\rangle = \frac{\left|00\right\rangle + \left|10\right\rangle}{\sqrt{2}} \quad (11)$$

Finally, when Hadamard gate is applied, $|\Psi_5>$ state is obtained as seen in Eq. (12):

$$\left|\psi_5\right\rangle = \frac{1}{2}\left(\left|00\right\rangle + \left|10\right\rangle - \left|00\right\rangle + \left|10\right\rangle\right) = \left|10\right\rangle \quad (12)$$

When the obtained value is examined, it is seen that a 10-information string is obtained with 100% probability.

If Alice wants to send 11 classic information, the iY operator is used and obtained state is $|\Psi_3>$ as seen in Eq. (13):

$$\left|\psi_3\right\rangle = \frac{-\left|10\right\rangle + \left|01\right\rangle}{\sqrt{2}} \quad (13)$$

When the CNOT gate is applied, $|\Psi_4>$ state is obtained as seen in Eq. (14):

$$\left|\psi_4\right\rangle = \frac{-\left|11\right\rangle + \left|01\right\rangle}{\sqrt{2}} \quad (14)$$

Finally, when Hadamard gate is applied, $|\Psi_5>$ state is obtained as seen in Eq. (15):

$$\left|\psi_5\right\rangle = \frac{1}{2}\left(\left|11\right\rangle - \left|01\right\rangle + \left|01\right\rangle + \left|11\right\rangle\right) = \left|11\right\rangle \quad (15)$$

When the obtained value is examined, it is seen that an 11-information string is obtained with 100% probability.

B. Teleportation Algorithm

Quantum teleportation is a technique of sending a state vector from one place to another. The distance between these two points is measured in light years. But the interesting thing is this. When sending a quantum state from one place to another, there is no need for any quantum channel between two points. Once it is possible to share classical information, it is possible to send a situation from one place to another. Again, when Alice and Bob were together years ago, they shared an entangled state (one of the Bell states) a qubit among themselves, Alice taking a qubit Bob. They are moving away from each other. Let one stay in our world and one go to another corner of the Milky Way galaxy. Years later, Alice wants to send a state vector to Bob. Alice and Bob cannot share any qubits between them, but they can share classical bits. In other words, it can communicate via a classical channel. This may seem impossible at first. The state vector of one qubit in Alice's hand is shown by Eq. (16) below [1]:

$$|\psi> = \alpha|0> + \beta|1\rangle \quad (16)$$

α and β in Eq. (1) are complex numbers that can take all values. It will take endless time for Bob to consider all possible values and construct them. Second, Alice cannot know the coefficients α and β because when Alice wants to measure, her state vector will be distorted. Therefore, it is unlikely that Alice will send Bob the state vector. But it is possible to send this state with a teleportation algorithm. Fig. 2 shows the quantum circuit of the teleportation algorithm.

After the CNOT, Hadamard, and measurement operators, the indications with two lines indicate that the communication is via classical channels. Those indicated by a line are quantum channels. After a measurement has taken place, a classical number (0 or 1) is obtained if a measurement is made on a qubit or a state vector. The state vector is destroyed and a real number is obtained. If there will be a transmission, it will be through the classical channel [1].

Years ago, Alice and Bob shared one of the Bell states among themselves. This is an entangled state, and this is one of the applications of quantum entanglement. The following Eq. (17) is an entangled state of two. The first qubit is Alice, the second qubit is Bob's. But the distance between them is very far.

$$\left|\beta_{00}\right\rangle = \frac{\left|00\right\rangle + \left|11\right\rangle}{\sqrt{2}} \quad (17)$$

It is an entangled state, so when one performs a measurement, the other is also affected. The situation $|\Psi>$ in Fig. 2 is the one that Alice wants to send to Bob. $|B_{00}>$ is one of the Bell states, that is, entangled states. Alice interacts with the situation she wants to send to Bob with her share of the qubit. Therefore, Alice has two qubits and Bob has only one qubit. Alice performs the measurement after first passing her qubit through the CNOT and then through the Hadamard gate. In Fig. 2, in the teleportation circuit, the general state that Alice

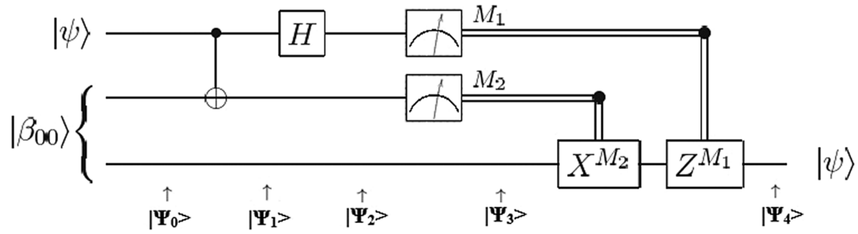

Fig. 2. Quantum circuit design of teleportation algorithm.

wants to send to Bob before passing to the CNOT gate and one of the Bell states they share among themselves are expressed as $|\Psi_1>$ in Eq. (18) below:

$$|\psi_1\rangle = |\psi\rangle|\beta_{00}\rangle = (\alpha|0> + \beta|1>)\frac{|00\rangle + |11\rangle}{\sqrt{2}} = \frac{\left[\alpha|000> + \alpha|011> + \beta|100> + \beta|111>\right]}{\sqrt{2}} \quad (18)$$

The first two qubits in Eq. (17) belong to Alice and the last qubit to Bob. After the $|\Psi_1>$ state vector is passed through the CNOT gate, the $|\Psi_2>$ state vector specified by Eq. (19) is obtained. The CNOT gate acts only on the first two qubits.

$$|\psi_2\rangle = \frac{\left[\alpha|000> + \alpha|011> + \beta|110> + \beta|101>\right]}{\sqrt{2}} \quad (19)$$

Then, when the first qubit of the $|\Psi_2>$ state vector passes through the Hadamard gate, when we put the common qubits in common brackets, the state vector $|\Psi_3>$ specified by Eq. (20) is obtained:

$$|\psi_3\rangle = \frac{1}{2}[|00\rangle\left(\alpha|0> + \beta|1>\right) + |01\rangle(\alpha|1> + \beta|0>) + |10\rangle(\alpha|0> - \beta|1>) + |11\rangle\left(\alpha|1> - \beta|0>\right)] \quad (20)$$

If Alice obtains 00 after performing the measurement, Bob does not need to do anything like Eq. (21):

$$00: Bob => \alpha|0> + \beta|1> \quad (21)$$

If Alice obtains 01 after performing the measurement, Bob should pass the state from X gate like Eq. (22):

$$01: Bob => \alpha|1> + \beta|0> \xrightarrow{X} \alpha|0> + \beta|1> \quad (22)$$

If Alice obtains 10 after performing the measurement, Bob should pass the state from the Z gate like Eq. (23):

$$10: Bob => \alpha|0> - \beta|1> \xrightarrow{Z} \alpha|0> + \beta|1> \quad (23)$$

If Alice obtains 11 after performing the measurement, Bob should pass the state first from the X gate then the Z gate like Eq. (24):

$$11: Bob => \alpha|1> - \beta|0> \xrightarrow{X} \alpha|0> - \beta|1> \xrightarrow{Z} \alpha|0> + \beta|1> \quad (24)$$

C. Bell's Inequality

Quantum mechanical logic is often difficult to understand because it violates classical logic. Bell's inequality is an inequality that was put forward concretely and numerically by the famous physicist John Stewart Bell in 1964 [12]. It was created with purely classical logic, which excludes the quantum mechanics system. And it has been revealed that this inequality is violated by the quantum mechanical system.

Bell's inequality indicates the results of measurements of three different bases on two particles as shown in Eq. (25):

$$\left|P(\vec{x},\vec{y}) - P(\vec{x},\vec{z})\right| - P(\vec{y},\vec{z}) \le 1 \quad (25)$$

In Eq. (25), the $P(\vec{x},\vec{y})$ indicates the mean value of the multiplied results by measuring the components of two entangled spin particles in the direction $\vec{x}$ and $\vec{y}$, respectively. The $P(\vec{x},\vec{z})$ and $P(\vec{y},\vec{z})$ values in Eq. (1) are valid for the $\vec{x}$, $\vec{z}$, and $\vec{y}$, $\vec{z}$ directions, respectively. $P(\vec{x},\vec{y})$ value is calculated by exploiting Eq. (26):

$$P(\vec{x},\vec{y})_{QM} = \langle\psi^-|\vec{\sigma}.\vec{x} \otimes \vec{\sigma}\vec{y}|\psi^-\rangle = -\cos\theta_{\vec{x},\vec{y}} \quad (26)$$

In Eq. (26), $-\cos\theta_{\vec{x},\vec{y}}$ indicates the angle between $\vec{x}$ and $\vec{y}$ as shown in Fig. 3.

If the particles are independent of each other, then the results of the measurements will indicate the inequality. If the particles are not independent of each other, in other words, they are entangled particles, then Bell's inequality will be violated.

To test Bell's inequality, it is assumed that Alice and Bob are long away from each other. The distance between Alice and Bob is assumed to be far enough apart that performing a measurement on one system can not have any effect on the result of measurements on the other. Alice and Bob's bit sequences must be entangled state. Then, we look at one of three possible directions x, y, and z of the spin of particles. If

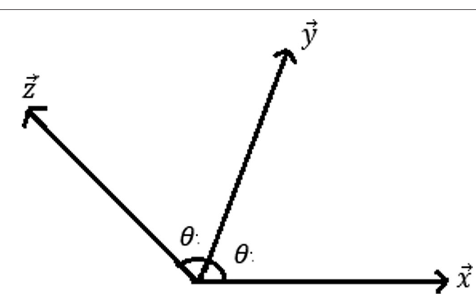

Fig. 3. Directions of the spin particles.

the revealed part of the sequence respects Bell's inequality, then we know that the qubits are not in an entangled state and they are acting like classical objects. If the revealed part of the sequence violates Bell's inequality, then we can assume that the whole sequence was measured when it was in a quantum entangled state.

III. EXPERIMENTAL RESULTS

A. Implementation and Results of Superdense Coding Algorithm

The circuit designs of superdense coding algorithm realized by using IBM Quantum Circuit Composer are shown in Fig. 4. In the first place, Hadamard and CNOT gate are added. Then, depending on the two-bit classical information to be sent, I, X, Z, and iY quantum gates are added to the circuit, respectively. In the next step, CNOT and Hadamard's gate are applied. Finally, since two bits of classical information were sent, measurement processing has been performed for both qubits.

When the circuit of superdense coding algorithm is executed on IBM real quantum computer which has 32 qubits with 1024 shots, obtained results for each 2-bit classical information are shown in Fig. 5 as a histogram. The results show that two-bit classical information is transferred with 100% probability by using a single qubit with superdense coding algorithm.

When superdense coding algorithm is executed on the simulation environment by using qiskit library, obtained results for each two-bit classical information are shown in Fig. 6 as histogram. The results show that two-bit classical information are transferred correctly by using a single qubit with superdense coding algorithm.

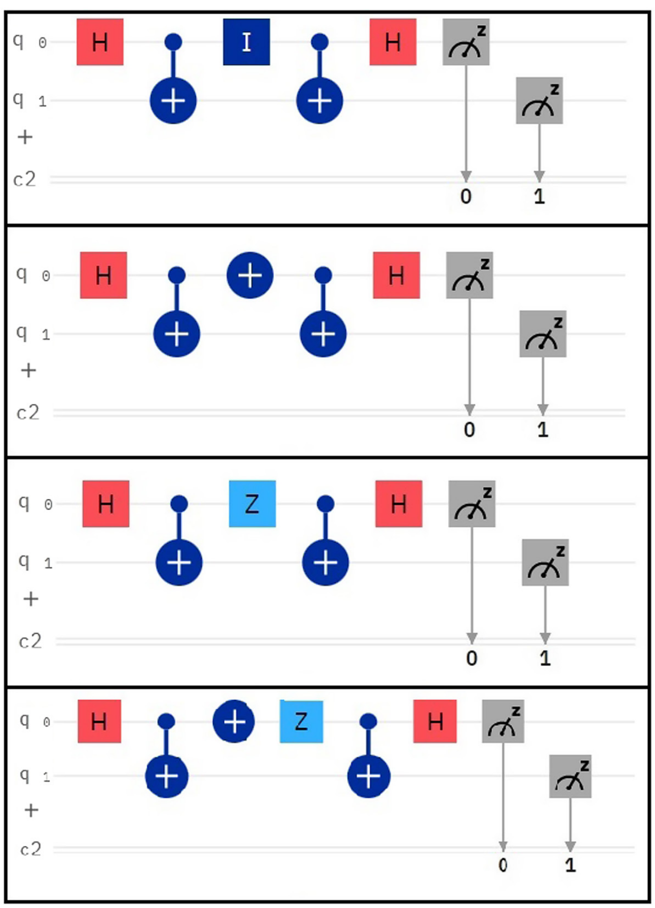

Fig. 4. The circuit designs of superdense coding for sending 00, 01, 10, and 11 classical information.

B. Implementation and Results of Teleportation Algorithm

The circuit design of the teleportation algorithm to perform the transfer operation of the |0> state by using IBM Quantum Circuit Composer is shown in Fig. 7. To implement the teleportation algorithm in IBM circuit composer, the first three-qubit system is composed. q_0 belongs to Alice. The purpose of the teleportation algorithm is to transfer the unknown state q_0 to q_2. First, the transfer operation of the |0> state is performed. To create superposition in other words entanglement state between q_1 and q_2, Hadamard and CNOT gate are added. Then, CNOT gate to q_0 and q_1, Hadamard gate to q_0 are applied and measurement operation is realized for both q_0 and q_1 qubits. In order to perform the transfer process of the q_0 state correctly, after the measurement process, CNOT gate between q_1 and q_2 and CZ gate between q_0 and q_2 are applied, respectively.

To perform the transfer operation of the |1> state, NOT gate must be added to q_0 qubit in the circuit as shown in Fig. 8.

In Fig. 9, the results of the transfer process of the |0> and |1> states on IBM real quantum computer which have 32 qubits with 1024 shots are shown as histogram, respectively.

When the teleportation algorithm is executed on the simulation environment by using Qiskit library, obtained results of the transfer process of the |0> and |1> states are shown in Fig. 10 as histogram, respectively.

Thus, in the measurement phase of the quantum teleportation circuit, Alice reconstructs the qubit that Alice wants to show herself by using different logic gates depending on the information received with a classical channel after Alice measures both her own qubits. This situation is called quantum teleportation or sending a state vector from one place to another. For this to happen, Alice and Bob must share an entanglement state among themselves. If there is no entangled state, it is not possible to send any state from one place to another.

C. Implementation and Results of Bell's Inequality Test

The singlet-spin state for the Bell's inequality test is shown in Eq. (27):

$$\left|\psi^-\right\rangle = \frac{\left|01\right\rangle - \left|10\right\rangle}{\sqrt{2}} \tag{27}$$

To create Bell state in Eq. (27), first, X gates are added to both q_0 and q_1 on the circuit, then Hadamard gate to q_0 and CNOT gate to q_0 and q_1 are applied. The violation of Bell's inequality with quantum mechanics are tested with two different angles. The angles $\Theta_{\vec{x}\vec{y}}$ between $\vec{x} - \vec{y}$ directions and $\Theta_{\vec{y}\vec{z}}$ $\vec{y} - \vec{z}$ directions are specified as $\frac{\pi}{4} = 45^0$ and $\frac{\pi}{3} = 60^0$, the angles $\Theta_{\vec{x}\vec{z}}$ between $\vec{x} - \vec{z}$ directions as $\frac{\pi}{2} = 90^0$ and $\frac{2\pi}{3} = 120^0$, respectively. Circuits of Bell's inequality test which are shown in Figs. 11, 12, and 13 measure the $P(\vec{x},\vec{y})$, $P(\vec{x},\vec{z})$, and $P(\vec{y},\vec{z})$, respectively, for the angles $\Theta_{\vec{x}\vec{y}} = 45^0$ − $\Theta_{\vec{y}\vec{z}} = 90^0$. Figs. 14, 15 and 16 show the circuits of Bell's inequality test for the angles $\Theta_{\vec{x}\vec{y}} = 60^0$ − $\Theta_{\vec{y}\vec{z}} = 120^0$.

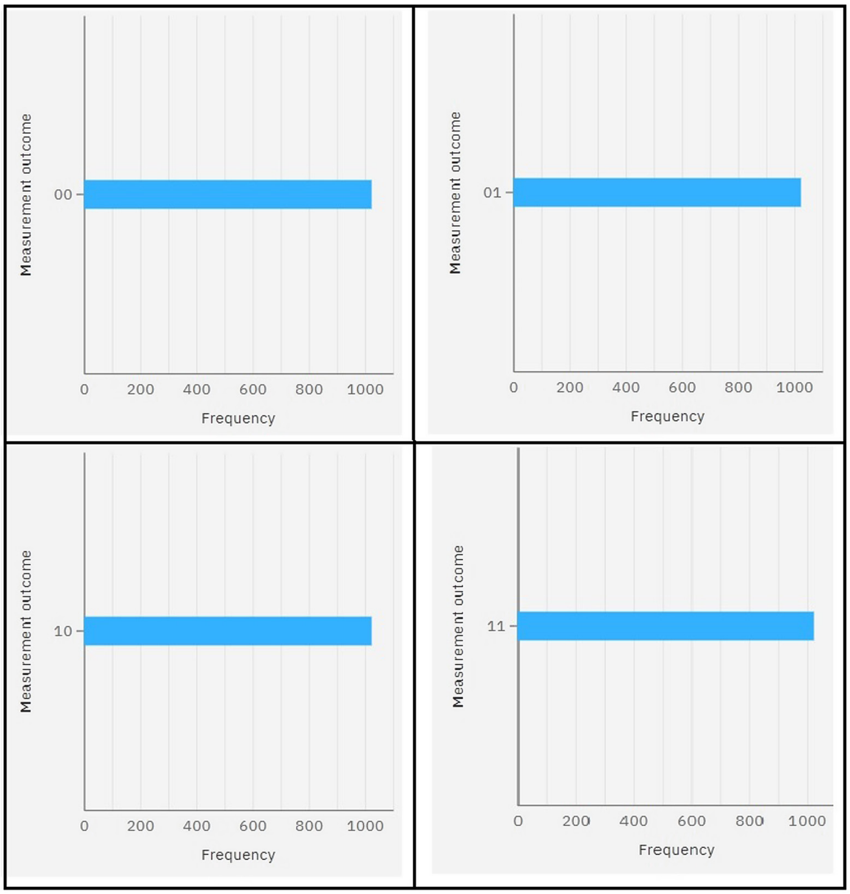

Fig. 5. Results of the superdense coding algorithm as a histogram on IBM real quantum devices. IBM, International Business Machines.

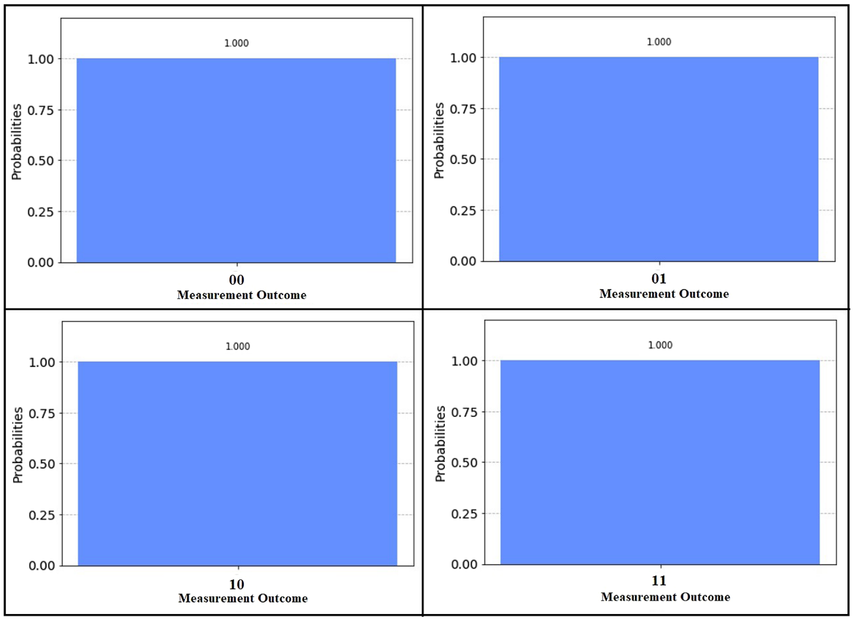

Fig. 6. Results of the superdense coding algorithm as a histogram on simulation environment by using Qiskit library.

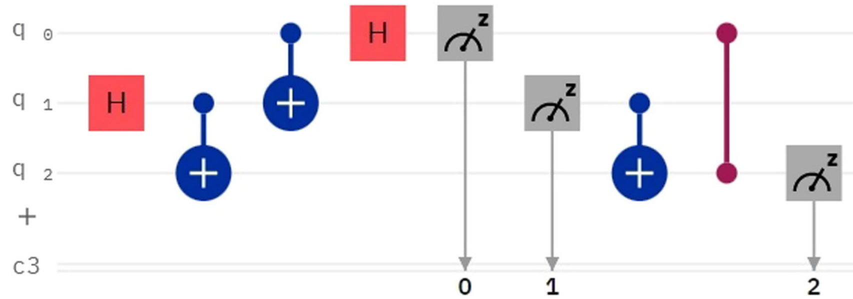

Fig. 7. The circuit design of teleportation algorithm to transfer |0> state.

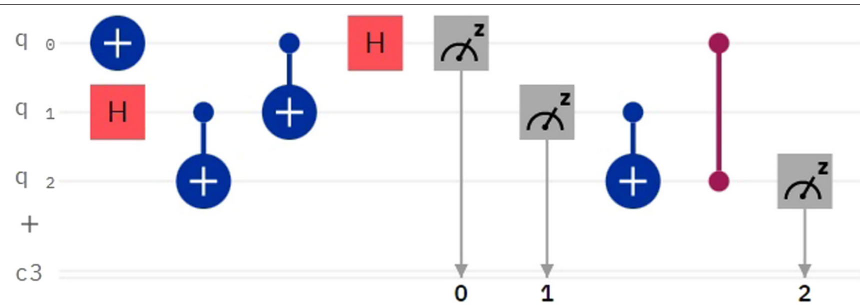

Fig. 8. The circuit design of teleportation algorithm to transfer |1> state.

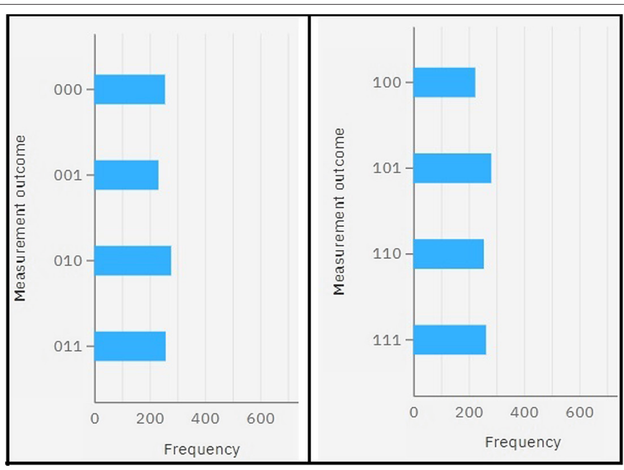

Fig. 9. Results transfer operation of |0> and |1> states exploiting teleportation algorithm as a histogram on IBM real quantum devices. IBM, International Business Machines.

In Figs. 17 and 18, the obtained results of the Bell's inequality test process for $P(\vec{x},\vec{y})$, $P(\vec{x},\vec{z})$, and $P(\vec{y},\vec{z})$ on IBM real quantum computer which have 32 qubits with 1024 shots are shown as histogram for the angles $\Theta_{\vec{x}\vec{y}}=45^0$ – $\Theta_{\vec{y}\vec{z}}=90^0$ and $\Theta_{\vec{x}\vec{y}}=60^0$ – $\Theta_{\vec{y}\vec{z}}=120^0$, respectively.

In Figs. 19 and 20, the obtained results of the Bell's inequality test process for $P(\vec{x},\vec{y})$, $P(\vec{x},\vec{z})$, and $P(\vec{y},\vec{z})$ on simulation environment by using Qiskit library are shown as histogram for the angles $\Theta_{\vec{x}\vec{y}}=45^0$ – $\Theta_{\vec{y}\vec{z}}=90^0$ and $\Theta_{\vec{x}\vec{y}}=60^0$ – $\Theta_{\vec{y}\vec{z}}=120^0$, respectively.

Experimental results for $P(\vec{x},\vec{y})$, $P(\vec{x},\vec{z})$, and $P(\vec{y},\vec{z})$ are presented in Tables I and II. $P(\vec{x},\vec{y})$, $P(\vec{x},\vec{z})$, and $P(\vec{y},\vec{z})$ are calculated by sum of probabilities of finding 00 and 11 minus probabilities of finding 01 and 10.

$$\underbrace{|-0.764-0.020|-(-0.690)\leq 1}_{1.474\leq 1} \tag{28}$$

$$\underbrace{|-0.674-0.040|-(-0.673)\leq 1}_{1.387\leq 1} \tag{29}$$

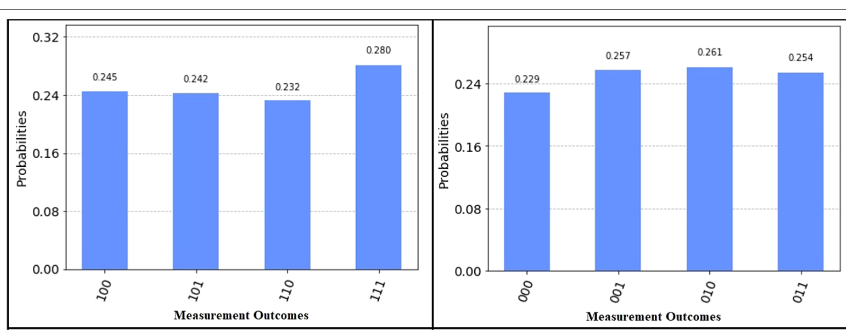

Fig. 10. Results transfer operation of |0> and |1> states exploiting teleportation algorithm as a histogram on simulation environment by using Qiskit library.

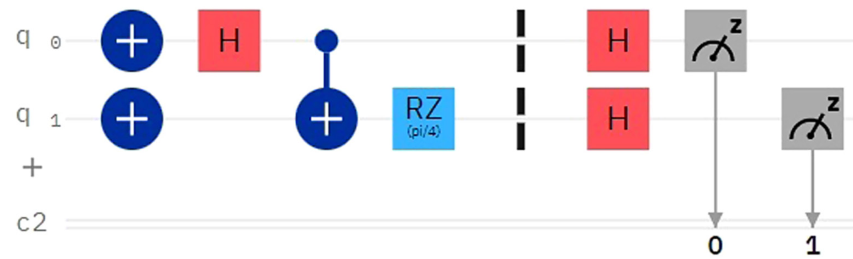

Fig. 11. The circuit design of Bell's inequality test for $P(\vec{x},\vec{y})$, $\Theta_{\vec{x}\vec{y}} = 45^0$.

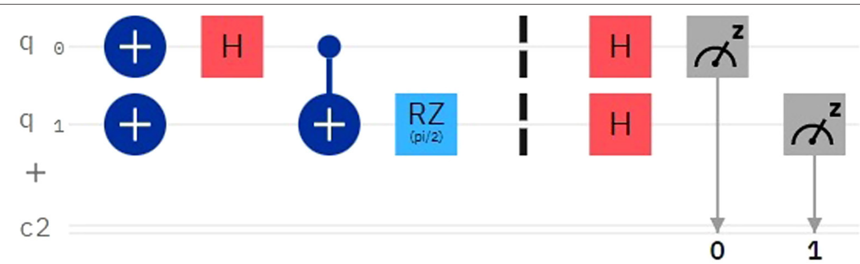

Fig. 12. The circuit design of Bell's inequality test for $P(\vec{x},\vec{z})$, $\Theta_{\vec{x}\vec{z}} = 45^0$.

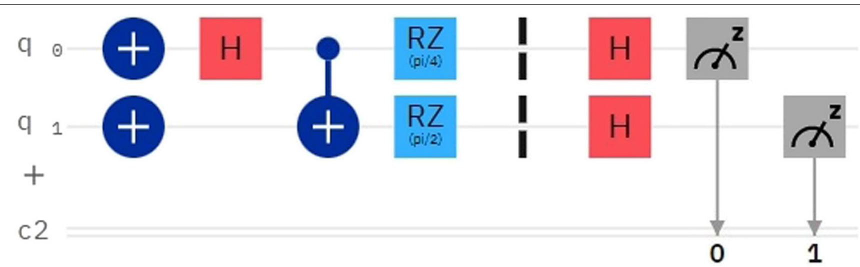

Fig. 13. The circuit design of Bell's inequality test for $P(\vec{y},\vec{z})$, $\Theta_{\vec{y}\vec{z}} = 90^0$.

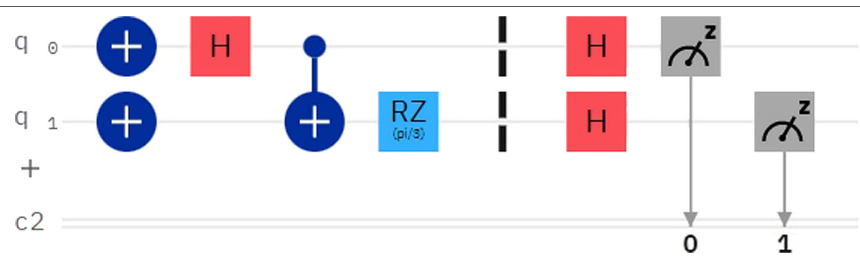

Fig. 14. The circuit design of Bell's inequality test for $P(\vec{x},\vec{y})$, $\Theta_{\vec{x}\vec{y}} = 60^0$.

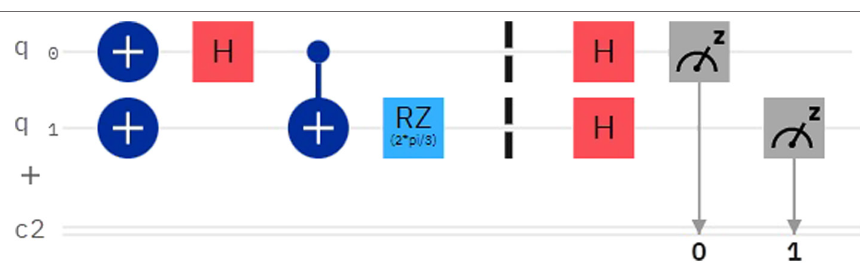

Fig. 15. The circuit design of Bell's inequality test for $P(\vec{x},\vec{z})$, $\Theta_{\vec{x}\vec{z}} = 60^0$.

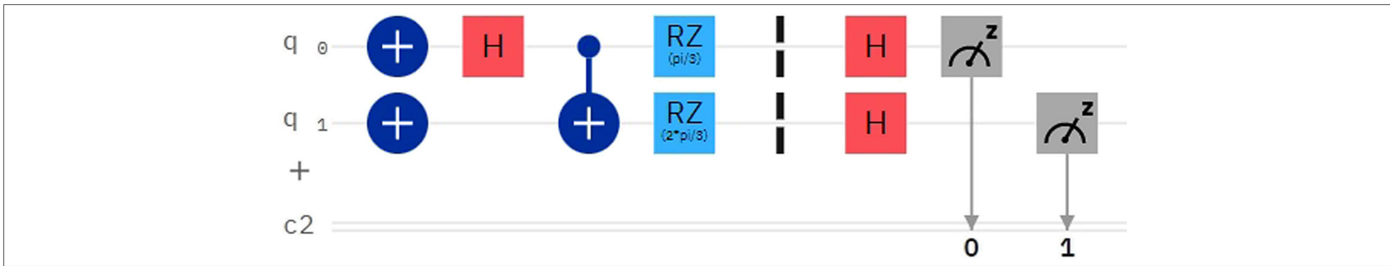

Fig. 16. The circuit design of Bell's inequality test for $P(\vec{y},\vec{z})$, $\Theta_{\vec{y}\vec{z}}=120^0$.

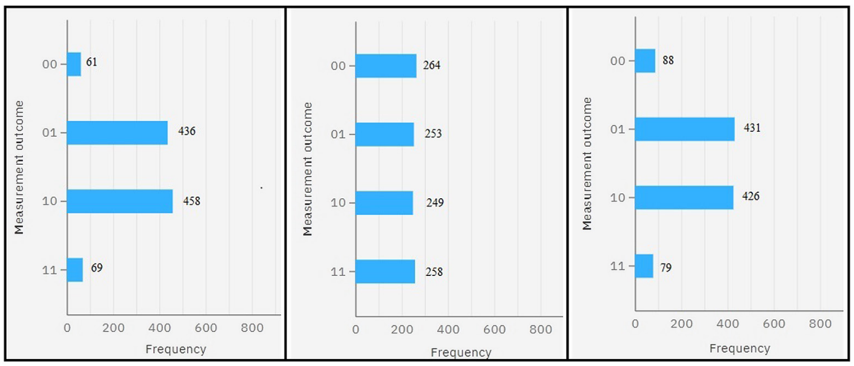

Fig. 17. Results of the Bell's inequality test for $P(\vec{x},\vec{y})$, $P(\vec{x},\vec{z})$ and $P(\vec{y},\vec{z})$ as a histogram on IBM real quantum devices for $\Theta_{\vec{x}\vec{y}}=45^0$ – $\Theta_{\vec{y}\vec{z}}=90^0$.

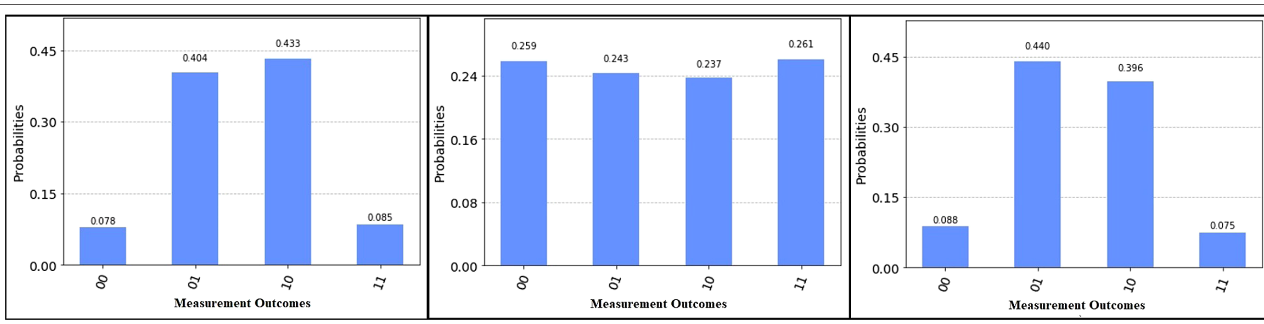

Fig. 18. Results of the Bell's inequality test for $P(\vec{x},\vec{y})$, $P(\vec{x},\vec{z})$, and $P(\vec{y},\vec{z})$ as histogram on simulation environment by using Qiskit library for $\Theta_{\vec{x}\vec{y}}=45^0$ – $\Theta_{\vec{y}\vec{z}}=90^0$.

$$\underbrace{\left|-0.472-0.494\right|-\left(-0.485\right)}_{1.450\leq 1}\leq 1 \tag{30}$$

$$\underbrace{\left|-0.475-0.498\right|-\left(-0.496\right)}_{1.469\leq 1}\leq 1 \tag{31}$$

When the values in Table I are placed in the Bell's inequality expressed in Eq. (25), the obtained results are shown in Eqs. (28) and (29) for the angles $\Theta_{\vec{x}\vec{y}}=45^0$ – $\Theta_{\vec{y}\vec{z}}=90^0$ on real quantum devices and simulation environment, respectively. When the values in Table II are placed in the Bell's inequality expressed in Eq. (25), the obtained results are shown in Eqs. (30) and (31) for the angles $\Theta_{\vec{x}\vec{y}}=60^0$ – $\Theta_{\vec{y}\vec{z}}=120^0$ on real quantum devices and simulation environment, respectively. It has been observed that Bell's inequality is violated by the quantum mechanical system both in the simulation environment and real quantum devices of IBM.

IV. CONCLUSION

In this paper, teleportation and superdense coding algorithms and Bell's inequality test are implemented in simulation environment by using the Qiskit library and on real quantum devices for the purpose of putting forward closeness of the results of running on simulation environment and real quantum devices. Our approach has proved that the results of the algorithms obtained on real quantum devices are very close to the results obtained in the simulation environment. Also, our approach to Bell's inequality has proved that the quantum mechanical system violates Bell's inequality.

Today, it is not possible to reach a speed higher than the speed of light. The quantum teleportation algorithm does not violate this law. Because in order for Bob to receive the state vector, Alice needs to send two bits of classical information. This information must reach Bob first so that Bob can construct his state vector. And these two bits of classical information are limited to the speed of light.

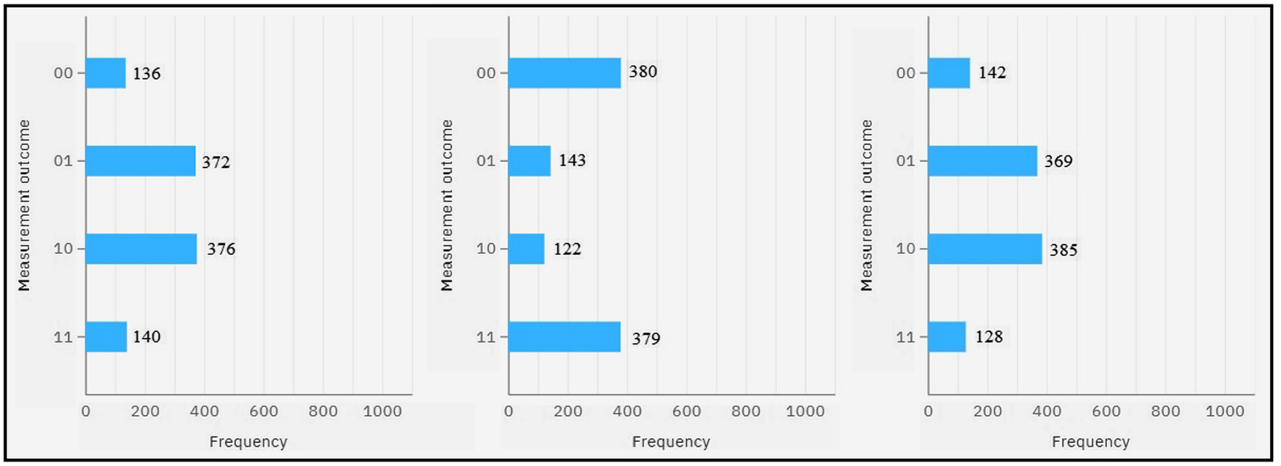

Fig. 19. Results of the Bell's inequality test for $P(\vec{x},\vec{y})$, $P(\vec{x},\vec{z})$, and $P(\vec{y},\vec{z})$ as histogram on IBM real quantum devices for $\Theta_{\vec{x}\vec{y}}=60^0$ – $\Theta_{\vec{y}\vec{z}}=120^0$. IBM, International Business Machines.

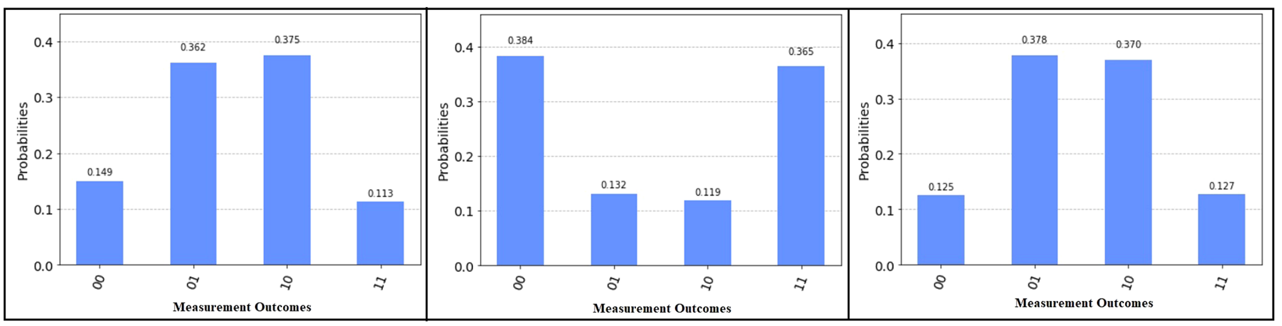

Fig. 20. Results of the Bell's inequality test for $P(\vec{x},\vec{y})$, $P(\vec{x},\vec{z})$, and $P(\vec{y},\vec{z})$ as a histogram on simulation environment by using Qiskit library for $\Theta_{\vec{x}\vec{y}}=60^0$ – $\Theta_{\vec{y}\vec{z}}=120^0$.

Copying a state vector in a quantum mechanical system is not possible according to the no-cloning theorem. This law has not been violated either. As Alice has measured her own qubits, one of these two qubits is his own and the other is an entangled state between them and Bob. When Alice made the measurement, she lost her state vector. Bob is reconstructing this state vector. Therefore, the state vector disappears in one place and reappears in another. Therefore, there is no irradiation. As a result, a state vector disappeared and reappeared [1].

In the future, quantum teleportation is foreseen to be used not only in the field of transportation and communication but also in the medical and military fields. The era of "virtual medicine" will begin in medicine. The codes of each organ will be copied and stored, and in case of damage to the organ as a result of a possible disease or accident in the future, the organ will be restored and treated using these codes. These technologies, which are currently only being put forward as fiction and theory, are getting closer to the desired level day by day [11].

TABLE I. EXPERIMENTAL RESULTS OF BELL'S INEQUALITY TEST ON IBM CIRCUIT COMPOSER AND SIMULATION ENVIRONMENT FOR $\Theta_{\vec{x}\vec{y}}=45^0$ – $\Theta_{\vec{y}\vec{z}}=90^0$

Directions	Results on IBM Circuit Composer	Results on Simulation Environment
$P(\vec{x},\vec{y})$	−0.764	−0.674
$P(\vec{x},\vec{z})$	0.020	0.040
$P(\vec{y},\vec{z})$	−0.690	−0.673

IBM, International Business Machines.

TABLE II. EXPERIMENTAL RESULTS OF BELL'S INEQUALITY TEST ON IBM CIRCUIT COMPOSER AND SIMULATION ENVIRONMENT FOR $\Theta_{\vec{x}\vec{y}}=60^0$ – $\Theta_{\vec{y}\vec{z}}=120^0$

Directions	Results on IBM Circuit Composer	Results on Simulation Environment
$P(\vec{x},\vec{y})$	−0.472	−0.475
$P(\vec{x},\vec{z})$	0.494	0.498
$P(\vec{y},\vec{z})$	−0.485	−0.496

IBM, International Business Machines.

Peer-review: Externally peer-reviewed.

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